Article

Multimodal analysis of videos based on the Theory of Objectification: contributions to research on mathematics teacher education

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Abstract
This paper aims to present and discuss the methodological contributions of the Theory of Objectification to multimodal analysis of videos in research on the training of mathematics teachers. In the first part, drawing on cultural-historical theoretical foundations and the Theory of Objectification (TO) and their implications for research, the use of videos is presented as a methodological strategy for capturing and analyzing data consistent with an understanding of learning as an embodied and material process occurring in collective sensible human activity. In the second part, examples of this type of analysis are presented and discussed in research data developed by the authors about the development of teachers' algebraic thinking. It is concluded that multimodal analysis involving images, oral and written records, gestures, etc., proposed by the TO — analyses in which different semiotic resources are investigated as a dialectical whole — offers a way to better understand the formative processes of teachers. By supporting the detailed analysis of the elements that impact the teachers' learning, it becomes evident how the awareness of the mathematical meaning at stake is developing in a process that, in turn, surpasses each register and its independent analysis. The transposition of the TO methodology to the training of teachers who teach mathematics is unprecedented, since this theory has been used to understand student learning in class. In the field of teacher education there is still much potential to be explored about the methodological approach based on the TO, which justifies the relevance of the discussion and proposal of video analysis presented in this text.

Keywords: Cultural-historical theory, Objectification processes, Multimodal analysis of activity, Mathematics teacher education.

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1. Introduction

The classroom as a living and, therefore, moving phenomenon is rich in diverse semiotic manifestations that are complexly expressed in speech, writing, gestures, corporal and facial expressions. The analysis of these semiotic manifestations can contribute to the understanding of the educational processes that come to life in this space. Such specificity makes the investigation of teaching and learning processes in the classroom a challenge for research in education and, in particular, for research in mathematics education.

Considering this complexity, the use of videos as a strategy to capture data from the reality of the classroom has been growing. It has been acknowledged that, "as a source of research data, video occupies an incomparable and possibly unique place in the mediation between the conception of real classroom practice and our ability to theorize about the characteristics of this practice" (CHAN, MESITI, CLARKE, 2019, p. 202). In addition, the facilitated access to technology and the accessibility of image capture equipment has allowed this data collection strategy to become more present in research, since the use of "videos makes it possible for intersemioses to be made between resources such as images, orality, gestures and sounds for the purpose of conveying an idea" (NEVES & BORBA, 2019, p. 221).

The way in which videos are used in classroom research is interrelated with the meaning that the video assumes in the capture and analysis of the phenomenon to be investigated, as well as in its subsequent analysis. Such meaning is produced in accordance with the theoretical understanding of the researcher about the phenomenon and is intrinsically related to the theoretical perspective that anchors the research. Thus, the metaphor itself to be used to portray the use of video is in a dialogue with the way in which the investigated phenomenon is understood in the theoretical perspective that underlies the research.

Understanding the need for a theoretical framework, we discuss in this text the use of videos in research on the training of teachers who teach mathematics, from a historical-cultural perspective. For this, we understand the training activity as a collective work or joint labour (RADFORD, 2016), or as a joint activity (RUBTSOV, 1996). In this sense, the video allows for the capture of the phenomenon in movement among the subjects of the formative process in a collective space. The study presented is the result of research conducted by the first author (V. M.) as a visiting professor in the *Laboratory of Research on Cultural Semiotics and Mathematical Thinking* at *École d'Éducation*, *Laurentian University*, which focused on the familiarization and understanding of data analysis methodology based on the Theory of Objectification (RADFORD, 2015).

The data analyzed using the TO methodology came from research carried out in Brazil and had the objective of investigating the development of the algebraic thinking of teachers in the early years when they were involved collectively in the resolution of situations that triggered the teaching of algebraic concepts. We emphasize that, although in this text we bring excerpts from this research, our objective here is not to explore the development of teachers' algebraic thinking, but to present and discuss the methodological contributions of the Theory of Objectification to research on the training of mathematics teachers. In particular, this text aims to discuss and analyze the contributions of the TO for

the multimodal analysis of videos produced in the context of research on the training processes of mathematics teachers.

With this objective, the text starts from aspects of the historical-cultural theoretical foundation and the Theory of Objectification and its implications for research on teaching and learning processes in the classroom. In the following, we present and discuss the use of video as a methodological strategy for capturing and analyzing data in research on the training of mathematics teachers, bringing some examples of that research which provided us with data for analysis according to the TO.

In general, the TO has been used to understand students' learning in the classroom, in a longitudinal way. The transposition of the TO methodology to the training of mathematics teachers is unprecedented; it opens the door for a methodological articulation with much potential to be explored, which justifies the relevance of the discussion and proposed analysis of videos that we present in this text.

2. Activity in the classroom as a multimodal phenomenon

The ways in which we understand classroom activity, the role of the teacher, and the importance or unimportance of the interactions between the individuals in that space, depend on the ontological and epistemological principles that underlie our conception of the individual and knowledge. Evidently, this is not a unique path and we do not intend to exhaust this discussion in this article.

For the discussion and analysis proposed in this article, we have studied and investigated classroom activity based on a historical-cultural understanding of the educational phenomenon. But what does this imply? First of all, it implies that knowledge is understood as a product of human activity occurring in a given historical and cultural context. Such human activity is not equivalent to simple disconnected actions, but activity presupposes a set of actions, articulated in an intentional way, which aim to respond to the motive that leads the subject to act (LEONTIEV, 1983). Activity also presupposes attitudes and social relationships between individuals (RADFORD, 2021a). In training contexts this activity is manifested when the subjects are involved in the resolution and discussion of proposed problems, not only because it was a demand of the teacher/teacher trainer, but because such proposed problems appear to them as motivating and, thus, the actions they develop seek to account for the objective of the activity.

A second fundamental aspect of the cultural-historical theory refers to the ontological principle according to which human beings are constituted in relation to each other, by means of semiotic and signified mediations. Vigotski understands that the psychic development of the subject is based on a dialectical movement of the higher psychic functions between the social and the psychic, and between the interpsychic and intrapsychic planes (VIGOTSKI, 1995). Thus, a historical-cultural approach assumes that "from a phylogenetic point of view, conceptual objects are generated in the course of human activity" which implies, in particular, that "[mathematical] knowledge is historically generated during the course of the mathematical activity of the individuals" (RADFORD, 2018b, p. 4069). From this perspective, learning can be understood as a process of appropriation of cultural meanings or, as some researchers prefer, as an

encounter with "historically constituted ways of thinking" (RADFORD, 2018b, p. 4067) in which students attribute personal meanings to such historical products of human activity, so that:

> learning is the reaching of a culturally-objective piece of knowledge that the students attain through a social process of *objectification* mediated by signs, language, artifacts, and social interaction as the students engage in cultural forms of reflecting and acting.

It is through different human activities that the subject meets this culturally and historically produced knowledge. Making this encounter viable, which we can understand as inherent to the learning process, orients the organization of pedagogical work. Thus, the school is the social place that is intentionally organized to guarantee the new generations an encounter with knowledge and ethical and aesthetic values that are understood by the collective as relevant or sufficient to be taught to the next generations (MOURA *et al*., 2010). In particular, the classroom is the privileged and intentionally organized space to put in motion this knowledge through the joint activity of students and teacher.

The collective or social aspect of human activity can be considered a third principle of a historical-cultural approach to the educational phenomenon and is related to the conception of the dialectical relationship between the social and the individual in the constitution of the human psyche (VYGOTSKI, 1995). The importance of the collective in the learning processes is defended by Rubtsov (1996, p. 134) who understands that learning takes place through "activity in common" characterized by the sharing of actions, exchanges in the modes of action among the subjects, mutual understanding of the relationships between the actions of the different participants, communication, planning and reflection that allows "going beyond the limits of individual actions in relation to the general scheme of activity" (RUBTSOV, 1996, p. 136).

Specifically, the importance of collective activity in learning processes, understood from the theoretical lens of the historical-cultural perspective, has also been emphasized in the teaching of mathematics. In this direction, Moura *et al.* (2010, p. 225) understands that "the solution of the problem-situation by the students must be done collectively. This occurs when individuals are provided with situations that require the sharing of actions in the resolution of a given situation that arises in a certain context".

The idea of collective activity is also central to Radford's theorizations (2015, 2016, 2021a, 2021b) by defining the concept of joint labour, or joint work, anchored in the concept of activity as the very movement of human life (MARX, 2015; LEONTIEV, 1983; RADFORD, 2021a, 2021b) in which we act, think, and feel in relation to others. "Activity (or joint labour with other people, in our terminology) includes language, but it also includes, in a decisive way, the embodied experiences of movement, action, rhythm, passion and sensation" (RADFORD, 2021a, p. 55).

Taking the classroom as a unit of analysis of the processes of objectification of knowledge in human activity, this joint labour, or joint work, is understood as a joint work of students and of teachers and students. "It is in the

production of this joint work that students are conceived of as gradually encountering and becoming aware of culturally and historically constituted forms of mathematical thinking" (RADFORD, 2021a, p. 55).

Joint labour is an essential category in the research and analysis proposed by the TO in its research on the problem of the development of algebraic thinking in the early years in dialogue with a historical-cultural perspective of knowledge. By longitudinally accompanying early years students in solving tasks in the introduction to algebra, the results of research have pointed out theoretical and methodological paths for a deeper understanding of this process (RADFORD, 2014; RADFORD, 2018a).

For this, it is necessary to return to the concepts of knowledge, knowing, and learning in a specific way, as understood by the TO. For Radford (2018a), knowledge is understood as a potentiality that people encounter through human activity. This encounter in this process of objectification of knowledge is called learning (MORETTI, PANOSSIAN and RADFORD, 2018). Thus, learning is understood as processes of encounter with cultural ways of thinking about the world in a perspective that understands human activity as essential in the process of objectification of knowledge. In addition to objectification, this encounter also results in the transformation of the individuals, by means of processes that the author calls subjectivations.

In the research on the development of the algebraic thinking of children from 7 to 12 years old, Radford relies on the dialectical materialist theory of objectification, understanding that "cognition can only be studied in movement" (*Ibidem*, p. 10) which converges with the principle of the Vygotskian method that understands that the basic requirement of the dialectical method is to study the phenomenon in movement, its stages and transformations, since "it is only in movement that a body shows what it is" (VIGOTSKI, 2002, p. 86).

For this, Radford (2018a) takes as a research focus the classroom activity, concentrated upon what he calls mathematical lessons. Classroom activity is essential in the perspective of the theory of objectification, since it allows the encounter of the individuals with knowledge (RADFORD, 2015). Activity understood as an ontological support for consciousness (LEONTIEV, 1983) is taken as a methodological unit of analysis in the theory of objectification.

When investigating the activity of subjects, the Theory of Objectivity does so by means of a multimodal analysis (RADFORD, 2018a) focused on the analysis of videos that allows us to follow gestures, expressions, language, and the use of different semiotic systems used to express mathematical ideas. In dialogue with the research by Radford (2018a) and Vergel (2018), it is understood that:

> The multimodal nature of human cognition means that we are not mere cognitive subjects. Our cognitive actions are always permeated by kinesthetic, perceptual, tactile, etc., and these sensory modalities are an integral part of the processes of cognition, they are not, it is clear, peripheral elements in the ways of knowing and working with mathematical ideas (VERGEL, 2018, p. 74).

The investigation of different semiotic systems used or produced by the individuals in dealing with mathematical situations sheds some light on the development of a way of thinking mathematically since each semiotic system reveals potentialities and limits in dealing with knowledge in general and, in particular, with mathematical knowledge since there are "always limits to what can be thought and said within a semiotic system. Each semiotic system has its own expressivity" (RADFORD, 2018a, p. 22).

As for the term multimodal, Sabena (2018) understands that, in the context of communication, it refers to different forms of communication and expression of meanings. Specifically, when investigating mathematical thinking and learning processes, Sabena understands that:

> multimodality refers to the importance and mutual coexistence of a variety of modalities or cognitive, material and perceptual resources in the processes of teaching-learning mathematics, and more generally in the formation of mathematical meanings (SABENA, 2018, p. 542).

The multimodal analysis of learning processes in the classroom has been a matter of investigation in Radford's research on the development of algebraic thinking. By investigating the processes related to the development of algebraic thinking in classroom activity, such research seeks evidence that demonstrates the awareness of the individuals about culturally constructed mathematical meanings. According to Radford,

> This [awareness] is empirically investigated, through the students' sensory actions, in perceptual, auditory, kinesthetic, gestural, linguistic and symbolic activity in general. This is why we investigate the multimodal activity of students and teachers (RADFORD, 2015, p. 560).

Thus, multimodal analysis can become an important ally in the production of a data analysis methodology consistent with an understanding of learning as a human activity and a process of awareness of the individuals.

3. Multimodal analysis of videos: The training of mathematics teachers as an object in motion

When analyzed from a historical-cultural perspective, teacher training practices can be understood as a collective activity, a collective work or joint labour (RADFORD, 2016), or even as a joint activity (RUBTSOV, 1996). Thus, when we investigate the continuing education of teachers in collective activity, taking as a methodological reference for analysis Radford's research on the processes of teaching and student learning, we understand that video recording allows us to capture the phenomenon in motion (VYGOTSKI, 1995) among the individuals involved in the training process.

Based on these theoretical and methodological assumptions, we initiated in 2018 a research project that aimed to investigate the development of the algebraic thinking of teachers in the early years when they were involved

collectively in the resolution of tasks elaborated as situations that triggered the teaching of algebraic concepts.

For the development of the proposed tasks, the researchers relied on the understanding that a TO task "is made up of a series of problems that students are asked to deal with" (RADFORD, 2021b, p. 173). As in Radford's research (RADFORD, 2021b), the elaboration of the task is the result of a joint work between professors, researchers, undergraduate and graduate students.

In this process, the starting point is a conceptual unit (in our case, the algebraic knowledge of the early years) and the problems consider a contextual unit that may involve, for example, a narrative history. Radford (2021b) weighs three elements in this task elaboration work: 1) general considerations (what students already know; use of artefacts); 2) considerations about mathematical problems (be interesting to students; allow for conceptual deepening; consider the conceptual and contextual unity and have increasing complexity); 3) Considerations on the organization of the classroom in order to potentiate the forms of human collaboration (encourage critical reflections; allow interactions between individuals).

Thus, in the research developed, we understand that a learning triggering situation (LTS) (MOURA *et al*., 2010) can be a task that, similarly, seeks to mobilize individuals in collective activity through the proposal of triggering problems in a context that is motivating for students. Specifically, in SDA, such objectives are considered by means of a triggering problem related to the historical human need that led to the production of the concept. For example, in the case of algebraic knowledge, we seek to consider situations that involve the need to recognize variable magnitudes and their relationships.

Based on this understanding, the work with teachers took the form of an extension course developed by researchers from the Grupo de Estudos e Pesquisas em Processos Educativos e Perspectiva Histórico-Cultural-(Geppedh-Mat)[3], of the Universidade Federal de São Paulo (Unifesp). We carried out a series of 20 meetings, developed in the school space, with the participation of 18 teachers from the initial years of elementary school. In the research data collection and analysis methodology, we sought to adapt and analyze the feasibility of the proposals presented by Radford (2015, 2018a) in classroom research with students for research in training spaces with teachers in continuing education.[4]

The researchers organized groups of four or five teachers from the initial years, who worked in different years, and who remained together throughout the meetings. The data collected included written productions of the subjects of the research and recordings of solutions of algebraic activities, as well as their audio and video recordings. Recordings were made of the work of each group, as well as of the group as a whole in moments of socialization. For recording the group work, a camera or cell phone and an audio recorder were used in each group in

[3] Grupo de Estudos e Pesquisas em Processos Educativos e Perspectiva Histórico-cultural. Line of research in Mathematics Education. More information about the group is available at the CAPES Research Groups Directorate, at http://dgp.cnpq.br/dgp/espelhogrupo/35714.

[4] The research originated from the first author's postdoctoral research on methodological appropriations of TO, under the supervision of the second author, at Laurentian University (Ontario, Canada) conducted between 2019 and 2020, as well as serving as a data base for two doctoral studies underway at Geppedh-Mat (Unifesp).

order to guarantee good quality audio. Subsequently, the audio and video files were synchronized into a single video file. In addition, all the sheets with the teachers' resolution records for the proposed tasks were digitized, as well as the field notes of the researchers.

This process resulted in a very large amount of data that were organized by dates and work groups. From there, we began the analysis process that aimed to identify stretches or excerpts in which the subjects demonstrate that they are becoming "progressively aware of culturally constituted mathematical meanings" (RADFORD, 2015, p. 560).

According to the methodology proposed by the Theory of Objectification (RADFORD, 2015), the unit of analysis assumed is the activity (LEONTIEV, 1983) in the classroom. This choice of the unit of analysis has as its theoretical basis the understanding of the dialectical unity between activity and consciousness.

> Although we come from a logocentric tradition, that is, a tradition that emphasizes the role of language and discourse in knowledge, we maintain that activity-based consciousness often arises at a sensory, preconceptual and pre-intentional level (RADFORD, 2015, p. 560).

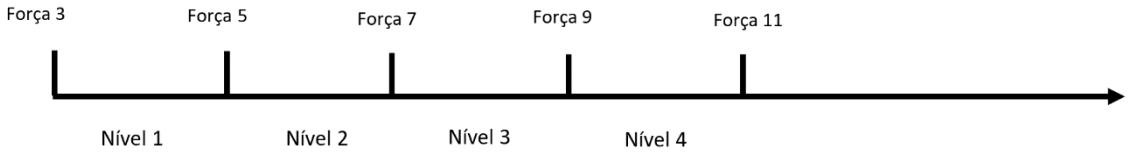
In line with this understanding, the TO proposes that the analysis of the data should have a multimodal character by considering as relevant data of the processes of objectification and subjectification, both oral and written language, as well as gestures, facial expressions, hesitations, etc.

In the case of our research, these semiotic manifestations were selected to the extent that they were representative of indications of the movement of development of the teachers' algebraic thinking. From the collected material and the understanding of the theoretical premises that should subsidize the analysis process, the analysis began with the transcription of the data and the selection of what Radford (2015) calls "salient segments". The salient segments are stretches of the data that bring some evidence or relationship with the object to be investigated.

Once the salient segments were selected, they were transcribed in detail, indicating the time of the beginning of the excerpt in relation to the whole video and adding observations that refer to gestures, expressions, pauses in the speech, etc. All these elements are recorded next to the transcription of the dialogues. In addition, images are selected to demonstrate the learning movement of the individuals in collective activity, or joint labour, in order to complement the analysis with the cross-checking of data from different semiotic registers.

In order to exemplify the use of the TO methodology in the analysis of videos in the context of training teachers who teach mathematics, we will discuss some research analysis excerpts on the development of algebraic thinking, at the moment when a group of teachers sought to generalize a situation that presented a game. In this game, the players started with the scoring of three "forces" and gained two "forces" at the end of each level reached, so that at the end of "Level 1" they had 5 forces, at the end of "Level 2" they had 7 forces, and so on, according to Figure 1:

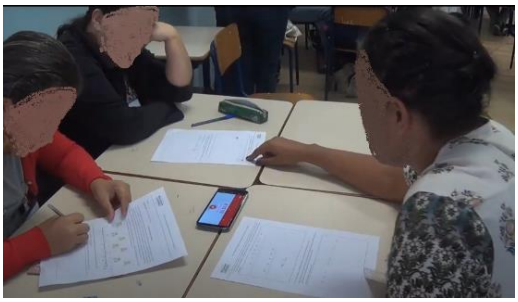
Figure 1 - Diagram of strength gain by level in the game presented to teachers



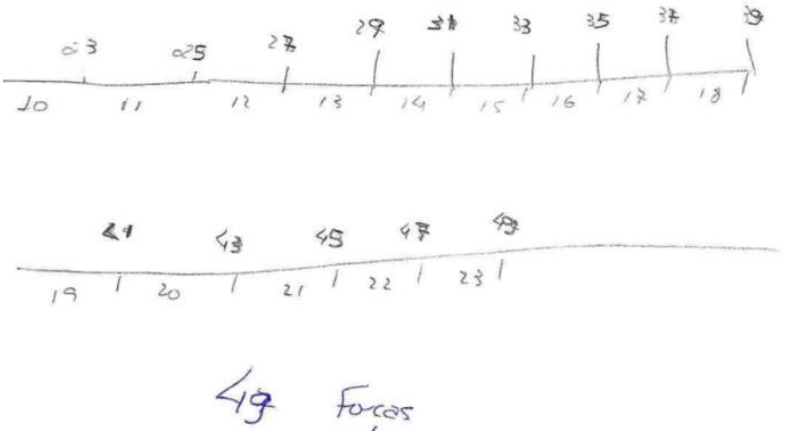
Fonte: Authors.

In the problem presented to the teachers, they were invited to help some children, who were supposed to be playing the game, to unravel the forces at each level. In the segment that begins at 13 minutes and 43 seconds of the video recorded in the third meeting of Group 4 (G4), the teachers Ema, Regina, and Carla discuss how to record the total calculation of the forces at each level (Table 1):

Table 1 - Meeting 3 of Group 4 – Episode at 13m43

	Transcrição do Episódio	**Interpretative Comments**
1.	 **Image 1:** Carla, Ema and Regina during Meeting 3.	Top left: Carla Bottom left: Ema Right: Regina
2.	*Ema: How can he* [the student] make a [semiotic] *representation of it? Go counting by twos, right?*	This refers to the possibility of calculating the forces for any level.
3.	*Regina: Yes. At each concluded level he gains two forces.*	First formulation of the awareness of a recurrent relationship: at the next level two forces are won.
4.	*Ema: Now I understood the reasoning, because I was counting the force. It is not to count the force, it is to count the level.*	Ema highlights the coexistence of two 2 mathematical variables: level and force. Thus, it contributes to a collective awareness of the mathematical relations underlying the problem.

5.	*Regina: At each level, two forces. For me, the only logic for a child to understand is that at each level she gains two. Always.* *Now, the value ... starting from three, I do not know.*  **Image 2:** Regina shows contrariness.  **Image 3:** Regina thinks for a while.	Image 2: Regina crosses her arms and turns her face, leaning her cheek on the palm of her hand, showing contrariness. Image 3: She turns her eyes to the left and remains silent, with her hand on her head, indicating that she is pensive. Regina draws the attention of her companions to the initial value of the sequence.
6.	*Regina: Ah ... unless you always place, to calculate the number of the level twice, plus one.*	Phrase uttered after a few seconds of silence. Regina proposes a first mathematical formula that relates the mathematical variables in the game.
7.	*Regina: Twenty-three, plus twenty-three ...*	Writing in the calculation sheet, the teacher is testing some particular values (Picture 4, below). Perception is accompanied and guided by mathematical signs, language, rhythm, and gestures.

	 **Image 4:** Regina's written record.	
8.	Carla: [It] is 47...	Accompanying the reasoning and indicating that the rule proposed by Regina does not reach the level result.
9.	Regina: Ah! It's the double of the level, plus the three initials.	Reformulation of the formula that recapitulates the collective sensible experience.
10.	Carla: Ahhh ... tá.	She stated that Regina's proposal made sense to her. This manifestation highlights the fundamental role of emotions in mathematical cognition.

Fonte: Authors.

In the excerpt transcribed in the table above, the crossing of the different semiotic registers allows us to recognize that when faced with the difficulty of finding a general solution to the problem (Line 5), Regina demonstrates contrariety (Image 2), which is followed by a moment of tension (Image 3) and a solution proposal (Line 6). In sequence, the testing of the proposal presented in Line 7 has as a reference the written record of the solution presented by the teacher (Image 4). The accompaniment of the oral, corporal, emotional, and written manifestation of Regina completes the analysis framework. It is important to emphasize that, despite the fact that in this study we focus the lens of analysis on Regina, her manifestations occur in collective activity through joint and mediated work with Ema and Carla. It is in the interaction with colleagues that there is the need to test hypotheses and reorganize arguments, which are validated or not by the group. Carla's statement in Line 8, when completing Regina's reasoning in testing the value of Level 23 (Line 7), shows that they worked together in search of the solution to the problem.

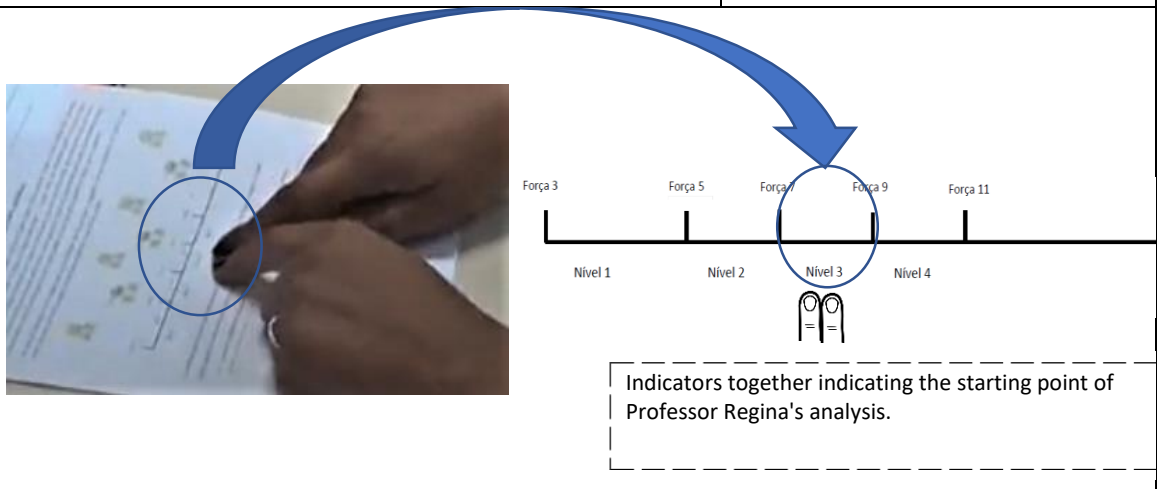
The excerpts presented in Tables 1 and 2 exemplify a path of multimodal data analysis based on TO. Thus, although the aim of this text is not to discuss

the development of the teachers' algebraic thinking, we have elaborated these tables as examples of the potential of data analysis based on the theoretical and methodological principles of the TO, since the perceptual register proposed in Table 2, which follows, complements the unit of analysis.

In Table 1, it is possible to identify that the oral and written records of professor Regina indicate a tension in the explanation of the relationship between level and force. She recognizes that strength results from the doubling of the level plus some value. In Line 6 she indicates that this value would be the number 1, and then (in Line 9), Regina indicates that the value to be subdued would be three, affirming "plus the three initials".

The perceptual (gestural) semiotic analysis presented in Table 2 helps us to understand how Regina overcomes this tension as she seeks to answer the question "but is that always the case?

Table 2 - Meeting 3 of Group 4 - Episode at 14m56s

	Transcrição do Episódio	**Interpretative Comments**
11.	*Regina: But is that always the case?For any level?*	A silence follows in the group. The question "*For any level*?" - expresses that now the problem is placed in general terms, involving now the infinity of possible numbers that the variable "Level" can take.
12.	*Regina: It's the same, or ... the level three times two, gives six.*	She talks while moving her fingers as shown in Image 5. Regina seeks through a concrete example to unravel the logic that can ensure the generalization.
	 **Image 5:** Image and scheme of Regina's gestures, indicating starting point.	
13.	*Regina: Plus the initial three, that's nine.*	

		At that moment, Regina moves the left indicator and points to the initial value of the forces.

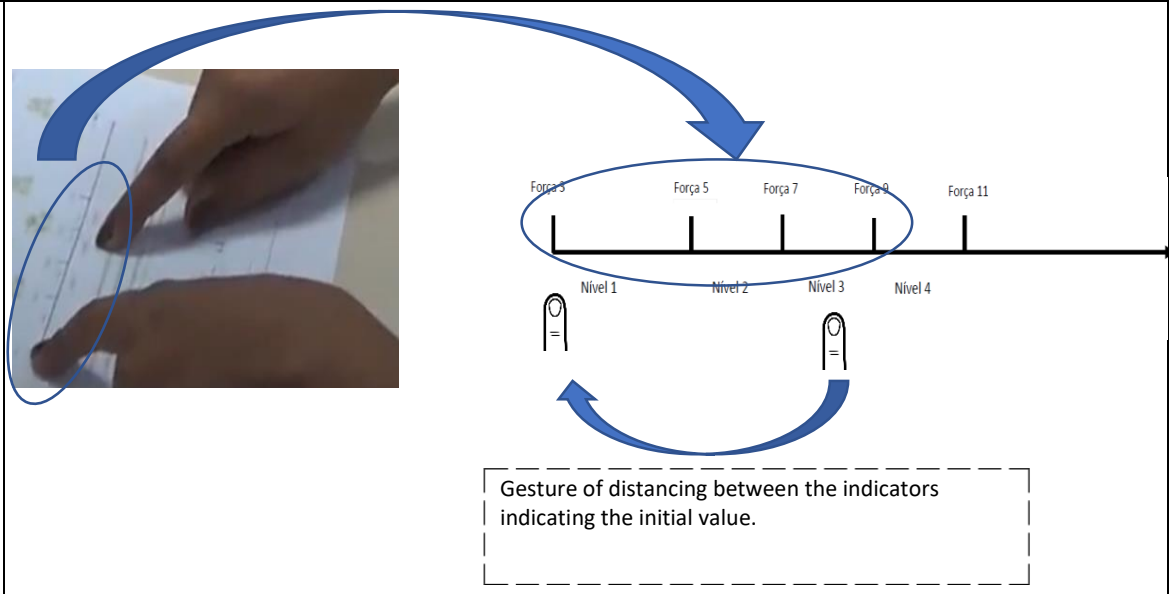


Image 6:
Image and scheme of Regina's gestures, indicating initial value to be submitted.

Fonte: Authors.

At the same time that she is oralizing the analysis of Level 3, Regina points the two indicators for the level (Figure 5). In sequence, she demonstrates recognition of the fixed value to be subdued (linear coefficient of the linear function) when she moves the left indicator to the initial value "three" (Image 6) at the same time that she states "Plus the initial three, it gives nine" (Line 13).

Thus, Frame 2 expands and qualifies the data analysis from Frame 1. This is because in Figure 2 there are detailed representative schemes of gestures that, when analyzed in unison with the other semiotic registers produced in the course of the teachers' joint work, allow us to recognize indicators of the process of objectification of algebraic knowledge in the collective activity of solving the proposed problem involving variation of magnitudes.

Thus, the semiotic analysis proposed by the TO allows us, in these excerpts taken as examples, to demonstrate that teachers produce and become aware, in collective activity, of a mathematical meaning that passes through diverse semiotic registers that complement each other. While the perceptual register (gestural) goes through a concrete dimension of the variable, the word expresses the possibility of generality.

5. Final considerations

In this text, we have presented and discussed some of the methodological contributions of the Theory of Objectification to classroom research and its potential for multimodal analysis of videos in research on the

training of mathematics teachers in the context of a Brazilian public school. For this purpose, we present part of the data that adapted the proposals presented by Radford (2015, 2018a).

The multimodal analysis proposed by the TO, by assuming as central the activity of the individuals and the joint labour that reveals the collective character of the educational processes, orients the crossing of different semiotic registers such as images, oral and written registers, gestures, etc. Such crossing of different semiotic registers produced by the individuals in collective activity, in joint labour, allows for a deeper understandinf of how the awareness of mathematical relations underlying the resolution of the given problem is taking place. The semiotic level of analysis opens a window to better understand the teacher training processes, analyzing in detail elements that impact teacher learning.

In this way of analyzing human activity, the different semiotic resources are not taken additively, but rather, they are analyzed as a *dialectical whole* whose manifestations allow a more complex understanding of the mathematical meaning at play, which, in turn, surpasses each register analyzed independently. Thus, it is possible to study "cognition in movement" (Luis Radford in MORETTI, PANOSSIAN and RADFORD, 2018, p. 4) in collective activity, which allows the encounter of the subject with mathematical knowledge.

The multimodal analysis of activity, based on the TO, when taken as a methodological strategy for the analysis of videos in research on teacher training, can bring important contributions to the understanding of teacher learning processes. This is because, by orienting the identification and analysis in a dialectical unity of perception, gestures, words, as well as oral and written registers, this form of analysis allows us to understand the subject as a whole in activity, revealing how the dynamic process of teachers' awareness of mathematical meanings occurs.

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