Article

Traces of school Mathematical Education in the theoretical matrices of the New School: from Herbart to Freinet

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Abstract
This is a paper that goes through matrices of the pedagogical thought of Modern Education, understood between the 17th and 18th centuries, and of Contemporary Education, referring to the 19th and 20th centuries, which constituted the New School movement. The manuscript is based on a literature review on the pedagogical ideas of Education theorists that make up these schools of thought. The purpose is to present in a panoramic way the pedagogical ideas of Modern Education, considering as a starting point the German educator Johann Friedrich Herbart and, as a finishing point, Contemporary Education, with the French educator Célestin Freinet. The aim is to identify the traces of school Mathematics Education woven by educators from the period considered. It is found that most of the pedagogical ideas of these Education theorists were mainly influenced by Jean-Jacques Rousseau, under the signs of the student's centrality, freedom and individuality and with the teacher conducting the educational process, using active methods. Regarding to school Mathematics Education, it was evidenced that the teacher develops his practice from very simple objects of the student's daily life, situations of their reality that can be questioned, educational toys, projects or workshops, so that, by making use of sensory perception or language, under an interdisciplinary perspective, it is possible to arrive at more elaborate ideas about mathematical concepts and/or contents.

Keywords: History of Education, New School, Math teaching.

1. Introduction

The present work proposes to approach aspects of the History of Mathematics Education present in classics of pedagogical thought, with emphasis on authors who thought, between the 18th and 20th centuries, the territory of Renewed Education. The purpose of this paper is to present, in a panoramic way, the pedagogical ideas of Modern and Contemporary Education, with a view to identifying traces of school Mathematics Education woven by educators from that period.

Clearly, thinking about Renewed Education is a complex and far-reaching task. Thus, the authors mentioned here were selected as some of the main exponents of the pedagogical debate that took place between the end of

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the 18th century and the 20th century, regarding a project to reconfigure school practices. Every selection, however, has some arbitrariness, although it is necessary to mention that it is understood that such theorists mobilized in this paper were representative of their time, influence current pedagogical practices, lead to reflection on Education in New School times and reverberate in discussions about Mathematics Education.

From a methodological point of view, this is a literature review that, according to Botelho, Cunha and Macedo (2011, p. 123), "requires the elaboration of a synthesis based on different topics, capable of creating a broad understanding of knowledge", and, in this paper, this review is constituted by the selection of eight, among the main theorists of New School, which are listed in Table 1.

Table 1 - Selection of the main educators at New School.

Educator	Nationality and life span	Academic background/occupation	Influences on your pedagogical ideas
Johann Friedrich Herbart	German 1776 to 1841	Law, dedicating himself to Philosophy later.	Locke, Kant, Rousseau and Pestalozzi (DALBOSCO, 2018).
Johann Heinrich Pestalozzi	Switzerland 1746 to 1827	He worked in political movements, he was a newspaper editor and ran an orphanage.	Rousseau, aspects of the romantic movement[2] and Kant (CAMBI, 1999).
Friedrich Wilhelm August Fröebel	German 1782 to 1852	He attended the services of his father, who was a Protestant pastor. He graduated in Surveying.	Rousseau, Pestalozzi, da filosofia da natureza de Friedrich Schelling (CAMBI, 1999), representative of German idealism.
John Dewey	United States of America 1859 to 1952	Doctorate in Philosophy.	Functional Psychology based on Darwin's Evolutionary Biology and William James' Pragmatism.
Maria Tecla Artemisia Montessori	Italy 1870 to 1952	Medicine. She founded the Children's House to assist the children of workers.	Rousseau and studies of French doctors Jean Itard[3] and Edouard Séguin.
Jean-Ovide Decroly	Belgium 1871 to 1932	Medicine. He founded the Ermitage Street School (ARANHA, 2012).	Rousseau; observations of concrete facts, practice of experimental methodology, founded schools-laboratories with a view to the psychological observation of children.
George Michael Kerschensteiner	German 1852 to 1932	School and university teacher, public school principal and politician.	Pestalozzi's educational philosophy, Dewey's sociological view of education and cultural-historical perspective of the

[2] A European movement that influenced culture and pedagogy (CAMBI, 1999), "preached the appreciation of feelings, intuition, inspiration and mystique, in contrast to the rationality of the Enlightenment" (ADORNO; MIGUEL, 2020, p. 2).

[3] Itard became famous for carrying out a study that "attempted to civilize the wild child found in the forests of Aveyron by stimulating and developing his senses" (RÖHRS, 2010a, p. 13).

			German philosopher Eduard Spranger.
Célestin Freinet	France 1896 to 1966	Herded sheep; incomplete teaching; active member of the union and the Communist Party.	His union and political activities influenced his own Pedagogy.

Source: Information extracted from the Educators Collection of the Ministry of Education (MEC) and authors whose references are shown in Table 1.

2. Modern Education: the systematization of pedagogical discourses

Modern Education is influenced by Descartes, who considers the idea of *cogito*[4] and reason as the starting point for all knowledge; by Bacon, who advocates valuing induction and experience; and by Locke, who, making use of the idea that the mind is a blank slate and that knowledge begins after experience, aims to mold the character of man, in view of the *gentleman* - gentle man (ARANHA, 2012).

In addition to rationalist and empiricist influences and the new society that was instituted, the way in which man realizes childhood in the 17th century changes, as it is realized in a negative view, therefore it becomes a moral concern for the adult, then the child needs to be policed and corrected. Faced with this feeling, the family resorts to the work of the religious and, subsequently, the need for the creation of schools arises (GAUTHIER, 2010).

Therefore, the creation of schools is not just for the elites, as with the increase of unemployed young people, which has become a problem for the bourgeoisie, and the concern with the correction of children, the school has a new use in society (GAUTHIER, 2010). Thus, a large part of the work that was carried out by religious is now implemented in schools, which cover a greater number of children and young people (CAMBI, 1999).

This new way of realizing childhood also contributed to the birth of Pedagogy, understood as “a set of rules [...] formulated for the master, in order to help him teach the student, so that he learns more, faster and better” (GAUTHIER, 2010, p. 126). The intent is to systematize the pedagogical dimension, seek precise methods and detailed education processes because, due to the increase in children and young people in school, questions have come to light on how to deal with large groups of students who remain in the same place and for a period continuous. Furthermore, as the skills of the masters of that time were transmitted to their successors, then a pedagogical tradition crystallizes, a way of knowing how to do what is called Traditional Pedagogy.

The pedagogues of the 17th century were inspired by nature for the constitution of the method, assuming that nature is harmonious and contrary to disorder and, in this context, Comenius proposes *The great didactics – Treatise on the universal art of teaching everything to everyone* (GAUTHIER, 2010), which makes him the first systematizer of the pedagogical discourse.

In this period, the traditional school was borning and, moreover, under the inspiration of rationalism and empiricism, that century was marked by the search for different methods that aspired to make education more pleasant and effective in practical life. This is Realism in Pedagogy: “To be realistic (from the

[4] This term is used in Descartes' maxim “I think, therefore I am” (DESCARTES, 1984, p. 22), *cogito ergo sum*. Thus, it is understood that the thinking subject is constituted from doubts.

Latin *res*, 'thing') means to privilege experience, things in the world and pay attention to the problems of the time" (ARANHA, 2012, p. 252).

Later, Étienne Condillac and Jean-Jacques Rousseau took up Locke's ideas and proposed other theoretical perspectives for the pedagogical field. Condillac understands that ideas are formed from sensations, their analysis and composition. Rousseau, on the other hand, presents the most original theory of the time, in such a way that it ends up renewing the entire pedagogical conception and praxis in force until that moment (CAMBI, 1999).

Rousseau stands out with his positive view of the child, with the elaboration of a theory of childhood whose objective is to understand who the child is and to materialize an educational program, albeit theoretical, that establishes practical rules to guide the work of educators. Through his work, the child becomes the center of education, so that there is a "Copernican revolution" in education. On the other hand, some aspects of Rousseau's thought are contrary to the discourses presented by the philosophers of Enlightenment[5], for this reason he is the target of attacks, as he is considered anti-rationalist and contrary to progress, "he opposed all of them current ideas (from tradition and from the century) in educational matters: from the use of diapers to 'reasoning' with children" (CAMBI, 1999, p. 343).

His work *Emílio*, published in 1762, provoked reactions among the leaders of the time, is presented as a treatise on "natural pedagogy" (MARTINEAU, 2010, p. 163) and generates numerous developments in educational theories of the time, reverberating even in the days of today. In this work, Rousseau brings out two fundamental principles for education: man is not a means, but an end and it is necessary to rediscover the natural man. Those who preceded Rousseau conceived the child as a means to reach a goal linked to a process of shaping the human being. However, from the Roussounian perspective, education must form man in his own essence; and not taking into account a particular "type of man" in view of a particular society or historical period. He understands that "education must imitate nature and follow the child's natural development in all points of view: affective, moral, intellectual" (MARTINEAU, 2010, p. 166).

According to this perspective, the child has a freedom that needs to be respected, it has its own nature that is different from that of the adult, it cannot be seen by the educator only as a passive being, but as an active being during learning. In this sense, the educator must provide conditions for the child to know for himself, through observation and experimentation.

> Rousseau therefore elaborates an **active pedagogy** (the child participates fully in the learning process), **concrete** (it resorts to observation), **essentially utilitarian** (it prepares for life among members in society), **centered on experimentation** and not on bookish study or in magisterial speeches (MARTINEAU, 2010, p. 169, bold author´s emphasis).

[5] The Enlightenment period refers to the intellectual movement that took place in the 18th century, and which was related to the triumph of reason for the Europe of knowledge. Man started to put his faith in science and progress, which, in turn, is linked to the idea that reason is beyond knowledge of the world, but also to act on it (MARTINEAU, 2010).

According to Martineau (2010), the fact that the educator intervenes as little as possible in the educational process provoked discourses contrary to Rousseau among pedagogues. However, his pedagogical idea tries to show that it is important to make the student feel the desire to learn for himself, that is, to take pleasure in his own instruction.

3. Contemporary Education and the New School: Rousseau's ideas remain

According to Saviani (2007), in the 17th, 18th and 19th centuries, teaching methods formulated from philosophical and didactic foundations are emphasized, while in the 20th century the emphasis shifts to learning methods, which establishes the primacy of psychological foundations of education.

In the nineteenth century, the German idealist philosophers Fichte and Hegel contradicted the Kantian idea that takes into account the use of reason to give form *a priori* to content arising from experience. They begin to consider the concept of human formation – *Bildung* – for an ideal of an integral man capable of reconciling sensitivity and reason within himself. Materialists, on the other hand, criticize idealism and influence the socialist perspective, especially the scientific one of Marx and Engels, as they understand that the action of educators – the struggle for a single school, the democratization of education and the awareness of the oppressed class, for example – can contribute to causing changes in the world while a revolution does not occur (ARANHA, 2012).

Another important point highlighted by the author is about the political character in education:

> [...] since the 18th century, the ideal of the secular, free and universal school, under the responsibility of the State, had already been outlined. Given its importance, education has increasingly taken on a political character, due to its role in society as an instrument for transmitting culture and forming citizenship: forming the citizen, that is, the political subject who knows his rights and duties (ARANHA, 2012, p. 421).

In this sense, since the eighteenth century the school was seen as a hope for the democratization of society. For this reason, some theorists of the 20th century criticized the school for inculcating ideals of the dominant class and, therefore, presenting an ideological character. On the other hand, the author stresses that at that time the policy was distorted to be used in education for indoctrination in countries with totalitarian Nazi, fascist, or Stalinist regimes, with the function of controlling children and young people.

However, despite these negative aspects in the educational context, the school also became the place for "kindergartens", for offering education to women, to the handicapped, and to the excluded ethnic groups. Moreover, according to Cambi (1999), the 20th century school opened itself to the masses and nurtured itself with ideology, even presenting a libertarian ideal.

In the 20th century, the Traditional Pedagogy, inspired by the industrial economic model, focused on increasing efficiency and with the schooling of the masses, is challenged by the New Pedagogy, also called New School (GAUTHIER, 2010).

The author says that one of the factors that cause this rupture from one Pedagogy to another is related to the scientific development that turns to the

needs of the child, aiming at a "new type of man". From then on, Pedagogy is understood as a practice that must relate to science in general and to child psychology. Moreover, it aims at a science of education based on observation and experimentation.

> In short, the new pedagogy places the child at the center of its concerns and opposes a pedagogy traditionally centered on the teacher and the content to be transmitted. This movement is the starting point of currents of thought that still exist today and that influence the set of current pedagogical practices (GAUTHIER, 2010, p. 175).

It is not possible to designate a single founder of the New School, although Claparède considers Rousseau as one of the great inspirers of this movement that brings together theorists of different nationalities from Europe and the United States. In any case, Gauthier (2010) argues that it is likely that the term new school has emerged in England, around 1889.

Chart 2 explains aspects that show the contrasts between Traditional and New Pedagogies.

Chart 2 - Aspects of Traditional and New Pedagogies.

Feature	Traditional Pedagogy	New Pedagogy
Terminology	Closed and formal pedagogy; encyclopedic; mechanical approach; passive school.	Open and informal pedagogy; active school; renewed.
Purpose of education	Modeling the child; transmitting objective values (the true, the beautiful, the good).	Develop the child's immanent strengths and personal values.
Method	Educating “from outside” to “inside”; the starting point revolves around the cultural aspects that must be absorbed by the child; pedagogy of effort.	Educating "from the inside" to "the outside"; the starting point is the child's personal aspect; pedagogy of interest.
Child's conception	The child is like clay that needs to be molded; it is necessary to act on it; the program is not for the child.	The child is like a creative energy; she acts; she is the protagonist.
Program design	The contents do not take into account the interests of the children.	Children's interests determine the program.
School design	It is an artificial medium; it represses emotions; it prepares for the future.	It is a natural and social medium; it helps in solving childhood problems.
The teacher´s role	Leader; it's in the center.	Guide, counselor, awakens to knowledge; the child is in the center.
Subject	Authoritarian, aims to coerce.	Attempts to promote student self-discipline.
Type of Pedagogy	From transmission; mechanism.	Child development; spontaneity.

Source: Adapted from Gauthier (2010, p. 194-195).

It appears that the Traditional and New Pedagogies are opposed to each other. In New School, the child is no longer considered as a reduced human being, but as a being different from the adult and who has its own characteristics, “it is perceived that the two great pedagogical trends lead us to a true theoretical aporia: both prove to be coherent and plausible, but apparently mutually exclusive” (SAVIANI, 2007, p. 105).

4. Mathematics teaching for character formation, educational toy and teaching materials: from Herbart to Montessori

In the history of Pedagogy, scholars who consider Herbart as a conservative theorist justify their position with the argument that he would have emphasized the directive role of the teacher and the contents, but would not have highlighted the active position of the student (DALBOSCO, 2018).

However, despite the centrality of the teacher in the Herbartian proposal, Neitzel (2015, p. 130) states that she "considers the child in its context, in its individuality, but giving fundamental importance to instruction and teaching". In this sense, Dalbosco (2018) understands that Herbart is the precursor of active methods and points to the important idea of self-government – "the child, properly governed by the adult, progressively achieves the ability to direct himself" (DALBOSCO, 2018, p. 8). This assertion underlies its Pedagogy.

Furthermore, the educational instruction is also central to Herbart's Pedagogy. It is an instruction whose objective is to educate, in the sense of transmitting new knowledge, improving pre-existing skills and favoring the emergence of capacities. Education in Herbart aims to shape human character through moral and civic education (HILGENHEGER, 2010).

The author argues that, from the Herbartian perspective, the child has a set of interests, but they need to be directed. Thus, instruction is based on the child's interests that are related to his or her previous learning experiences. In this sense, instruction improves preexisting knowledge.

Lourenço Filho (2010) states that educational instruction is organized in phases: clarity in the presentation of the subject, association, systematization and application. In the initial phase of the class, the teacher must consider aspects of the reality of the environment, in the second moment, bring notions from previous classes, in order to develop the student's apperceptive capacity. In the third phase, the student must be led from isolated images to the organization of concepts, by increasing generalization, which occurs when noticing similarities and differences. Finally, this knowledge is applied to practical situations.

That said, Hilgenheger (2010) points out that educational instruction distinguishes Herbart's Pedagogy from Traditional Pedagogy, whose main objective is to inculcate a large amount of information in the student; in Herbart, one of the aims of teaching mathematics was to contribute to the formation of the student's character. In his 1802 treatise, Pestalozzi's *The Idea of an ABC of Intuition*, he outlines an ultramodern initiation program for Mathematics for the time, in which he talks about how the teaching of Mathematics should contribute to Education, not only the practical utility or technological importance of Mathematics (HILGENHEGER, 2010).

The teaching of Mathematics also comprises Geometry, Algebra, Theory of Logarithms and Differential and Integral Calculus, and according to Kang (2012), as well as the teaching of Natural Sciences, that of Mathematics began with perception exercises, mediated by sensory experience. In this way, it appears that his ideas regarding Mathematics Education were inspired by Pestalozzi, which leads us to understand that perception is constituted as a representation obtained from the real, so that it is an idea that forms the educational content.

While Herbart values the instruction, Pestalozzi understands that education is superior and that is "a process that should follow nature, freedom,

the innate goodness of the human being, uniting mind, heart and hands[6]" (SOËTARD, 2010, p. 35), and such a trilogy should be kept in balance in the teaching of any school subject.

Pestalozzi considers the child as the center of the educational process, so that his method is based on the knowledge of the child, adapted to his level of development, advocating child spontaneity, which must be preserved. In this process, the teacher assumes the role of guide (ARCE, 2002 *apud* GASPARIN, 2010).

In addition, he mobilizes the idea of intuition for the educational field, which is why his Pedagogy is called Intuitive, as it considers the senses as a means for structuring the mind. From this perspective, intuition underlies human knowledge and, therefore, instruction (SOËTARD, 2010).

In Oliveira (2017) it is understood that, for Pestalozzi, the activity of the human spirit is manifested through intuition:

> Intuition for Pestalozzi was not limited only to manifesting the impressions of things, but also to driving the first actions of intelligence. As I understand it, sensitive intuition has been elevated to the level of the *art of intuition* characterized as the judgment and reasoning of received impressions. Years later, in 1826 to be more precise, Pestalozzi called this *art of intuition* inner intuition (OLIVEIRA, 2017, p. 1010, italic author's emphasis).

Pestalozzi's didactic method can be characterized as an active process in which knowledge is initially constituted by practice that occurs through perception and sensory experience and, later, arrives at thought (GASPARIN, 2010).

> The method [...] should start from the known to the unknown; walking from the concrete to the abstract; get the child used to doing; not telling the child what he can discover for himself; follow the order of nature; direct the mind and senses from the particular to the general, moving from intuitive vision to general understanding, developing in students the ability to perceive and observe (GASPARIN, 2010, p. 35).

Pestalozzi designates number, form and language as elements of the genesis of knowledge. According to Ferreira and Santos (2018), his intuitive method – based on intuition and perception – is characterized as a process of child development as a whole and which starts from these principles: number, form and word, in addition to its relationship with faculties and human capabilities.

Numbers are considered as abstractions, so they must be preceded or followed by arithmetic with initial design fundamentals. By using the student's intuition and "from the concrete to the abstract", the numerical operations are carried out by the use of real objects (PESTALOZZI, 2010). Thus, in relation to the teaching of Mathematics, children could count objects from a collection through imitation when watching the adult count (NETA; GUTIERRE, 2020); on

[6] They are the three basic aspects of the organism: intellectual, moral and physical (GASPARIN, 2010). In this trilogy, the head represents the power of man arising from his capacity for reflection and elaboration of concepts, the heart is the dimension that is linked to his sensitivity and empirical experience in the world to dominate nature through work, the hand being the dimension linked to "doing", given its empirical experience (SOËTARD, 2010).

the other hand, the notions of quantities could also be taught through the presentation of objects to the child, so that they could associate them with the quantities they represented (FERREIRA; SANTOS, 2018).

Then, there is an emphasis on the presentation of objects in front of the child, which occurred, according to Ferreira and Santos (2018), for the teaching of Geometry - in the case of geometric solids, to establish relationships between measurement and drawing.

As for the teaching of numbers, the use of syllable tables is considered, which combined the learning of numbers with that of words, so the child was asked how many syllables in the words were and arithmetic operations were performed. Geometry was studied through form, measurement and design (NETA; GUTIERRE, 2020).

Fröebel, like Pestalozzi, integrates head, heart and hands. Their education is active, centered on experience, advocating the freedom of the child and the education of their body, so it is important for the teacher to correct the child's body (HEILAND, 2010).

According to Heiland (2010), his Pedagogy is based on the family model, so the idea of family – an environment with love, peace and care – should be applied to educational institutions, which he calls communities, where they should even be vegetables were planted that would be used to feed the students, and the animals should also be taken care of by them. According to the author, in Fröebel's conception, children, as well as fauna and flora, develop naturally and in a harmonious way, so he brings up the idea of kindergarten gardeners, such as gardeners who take care of flowers.

He argues that learning should take place in early childhood, before the age of seven, so he creates kindergartens - *kindergarten* -, intended for children aged 3 to 7, "where professional educators (*kindergarten*) take care of children. little ones making them play" (HEILAND, 2010, p. 31, italic author´s emphasis).

Fröebel also elaborates the Pedagogy of Games, in which the games, called gifts, are an initiation to a kind of work, therefore, a relationship between working and playing is verified. In this context, the rules of the game must be well understood to favor the participation of all and the adult gives support, in the sense of symbolizing the experience, that is, to attribute meaning to it, "bringing to the child's consciousness" what can be learned through the experience resulting from the game (FROEBEL, 2010).

In relation to Mathematics, he advocates that without it, teaching would not be complete, since it is the teaching of Mathematics that would allow the full development of the human spirit (FROEBEL, 2010). In his teaching perspective, there is a centrality in the empirical for the elaboration of mathematical concepts or contents, because, according to Heiland (2010, p. 26), Fröebel admits that man is capable “of 'mathematically' structuring the real to extract significant relationships from it.

Like Pestalozzi, Fröebel considers language as an instrument to designate reality and intellectual productivity. In this sense, it is understood that “mathematically structuring the real” takes place through mathematical language, that is, the real is conceived through mathematical language. He understands that Mathematics is found in real objects, figures and nature – in the empirical – and manifests itself through human thought.

Another aspect that is observed in Pestalozzi and in Fröebel is learning through observation and imitation of what parents or educators do, whether for

domestic activities or even for Mathematics, such as counting objects, for example. From his point of view, the series of numbers need to be taught clearly, correctly; and not in the form of a mechanical repetition. Thus, it highlights the importance of the child having the objects in front of him for counting, so that he can relate the name of the numbers to the quantities (FROEBEL, 2010).

That said, according to Reinhold *et al.* (2017), like Pestalozzi, Fröebel was convinced of the insertion of concrete objects for educational use. Thus, in kindergarten, blocks were used for the understanding of Geometry in the early years of life, to explore spatial relationships or properties of Geometric Solids. Regarding the teaching of Geometry, he maintains that its beginning should be given by objects with straight lines, and then the curvilinear ones can be worked on. Subsequently, lines in the plane, angles, surfaces, part-whole and whole part relationships, for example, can be explored (FROEBEL, 2001).

The six educational toys (gifts) developed by him were used in different contexts, including for teaching Geometry in Primary Mathematics Education. The first includes six balls of different materials, the second presents a variety of solids – cube, cylinder and sphere – and the others are composed of solids that allow their decomposition into smaller cubes or rectangular or triangular prisms of different sizes (REINHOLD *et al.*, 2017).

> By arranging the pieces in this way, a child explores what Fröebel called *forms of knowledge* (dividing the entire cube into smaller and equal pieces), *forms of life* (arrangements that resemble objects from a child's environment such as a sofa or stairs) and *forms of beauty* (symmetrical arrangements of smaller pieces). This concept by Fröebel aligns with Pestalozzi's philosophy, reflecting three different approaches to life and learning: *Forms of knowledge* - our head and mind; *life forms* - what we touch with our hands; and *forms of beauty* - what touches our heart (REINHOLD *et al.*, 2017, p. 436, italic author's emphasis, my translation).

With regard to Montessori, Röhrs (2010a) explains that his educational precepts are worldwide. It aims at a science of education, as it provides Early Childhood Education with a scientific basis based on experiences and precise observations on child development. His education advocates the independence of the child, so that he can act freely, because in this way the development of his superior forms and functions occurs faster and more perfectly.

For this, Montessori understands that an appropriate environment is necessary for the child to live and learn: “When we speak of 'environment', we refer to the total set of things that the child can freely choose and handle to satiety, according to their trends and impulses of activity” (MONTESSORI, 2010a, p. 65). In this sense, the furniture needs to be proportionate to the child, so that it is accessible to him.

As the child must be free to move, discipline must be active, the individual “is master of himself” (MONTESSORI, 2010a, p. 69), but his freedom is limited by the collective interest.

From the Montessori point of view, pedagogical activity should help the child to advance on the path of independence and for him to be self-sufficient (RÖHRS, 2010a). In addition, the child's freedom is valued, which does not mean disorderly external acts or leaving the child abandoned, but "it is about 'freeing'

the child from obstacles that prevent the normal development of his life" (MONTESSORI, 2010a, p. 73).

Other important aspects are sensory education, so that with the different sensory stimuli, sensitivity and organization are fine-tuned, “The child cannot live in disorder because it causes him suffering that is manifested through desperate crying and even of a flurry persistent disease that can assume the appearance of a true disease” (MONTESSORI, 2010b, p. 109).

According to Neta and Gutierre (2020), the Montessori method starts from the concrete to the abstract, so the educator developed adequate teaching materials to provoke the child's reasoning, through the teacher's mediation.

Montessori teaching material should favor abstraction and encourage generalization. To the extent that the child is in contact with this material, he responds to serious work that requires his concentration, “It really seems that children are reaching the greatest achievement of which their spirits are capable: the material opens to intelligence ways that, at that age, would be inaccessible without it” (MONTESSORI, 1969, p. 197-198 *apud* RÖHRS, 2010a, p. 23).

Montessori also contributed to children's Mathematics Education, mainly with the publication of the works *Psychogeometry* and *Psychoarithmetry*, both in 1934, which discuss the teaching of Mathematics and pedagogical materials (COSTA, 2001 *apud* NETA; GUTIERRE, 2020).

For the counting of objects in early childhood, the educator suggests that the process occurs gradually, starting with a few objects and increasing the amount in a subtle way. In this process, it is also possible to recognize tactile differences in relation to the surface of objects, compare their masses, shapes, work with colors and noises. It is not about analyzing the objects, as it would be premature for early childhood, but it is about observing them in a way that it is not necessary to enter the domain of the concepts of Geometry, for example (MONTESSORI, 2010a).

Montessori, like the educators already presented in the text, does not dissociate the use of concrete materials from sensory education for the teaching of Mathematics. Silva and Soares (2020) mention that sensory education prepares children for calculation, as it allows them to become familiar with the notions of quantity, size, equality and difference. In addition, Montessori teaching material makes it possible to group objects and fit cylinders according to a series. The tablets divided into centimeters make it possible to establish the first notions of numbers and their juxtaposition favors the children to arrive at the ideas of addition and subtraction. Size comparison games introduce the concepts of width, length and thickness; cards with numbers cut in sandpaper and other smooth ones can be used for counting, associating and remembering numbers, notion of pairs, dozens and tens and arithmetic operations initiated with the length scale, for example.

Neta and Gutierre (2020, p. 17) mention that the Golden Material was also developed by Montessori, “consisting of 500 cubes measuring 1 cm x 1 cm x 1 cm, which represents the unit; 100 bars of 1 cm x 10 cm x 1 cm, representing the ten; 10 plates of 1 cm x 10 cm x 10 cm and 1 cube of 10 cm x 10 cm x 10 cm, which represents the unit of a thousand”, through which the place value of the digits could be worked out and establish relationships between them. In addition, the Golden Material is still being used to work on the structures of the Decimal Numbering System, the four operations, geometric concepts, fractions, decimal numbers, percentage, areas and volumes.

5. From Project Pedagogy to the School of Work: from Dewey to Freinet

Dewey's Pedagogy, like the educators mentioned in the previous section, is student-centered, aims at the student's performance and the exercise of democracy through the school experience - the school is "an experiment in education for democracy" (WESTBROOK, 2010, p. 26). Dewey defends the school linked to social progress and becomes a reference for the active school movement (CAMBI, 1999).

Dewey understands that knowledge occurs through the confrontation and resolution of problem-situations by the student. In this context, it is up to the teacher to identify the student's previous knowledge and interests, what he brings from his home or surroundings, for example. Thus, the teacher can use what the student brings to guide their practice. Thus, it appears that Dewey's Pedagogy is based on North American pragmatism, in which teaching should be related to useful knowledge for the student's life. He postulates that it would be necessary to relate the needs of the student to the curriculum (WESTBROOK, 2010), so that "It is absurd for the teacher to establish the 'own' objectives as objects suitable for the development of the students, in the same way that the farmer would be. set an agricultural ideal regardless of actual conditions" (DEWEY, 2010, p. 81).

The children were divided into groups according to their age to work with projects related to a topic of interest to them and the scientific method was used to use the students' experiences, which, according to Andrade (2011, p. 124), consists of the "definition problem, suggestion of a solution, development and application of the experimental test, and formulation of the conclusion". Therefore, from Dewey's perspective, education gains meaning through experience, knowledge is a problem to be solved and the student's interest needs to be valued.

According to Westbrook (2010), Dewey called occupation an activity performed by the child that reproduces a type of work in society. The occupation related to building a model of a farm, for example, allowed students to learn notions of measurement units and fractions, based the study of these mathematical contents and familiarized them with problem solving. Dewey's Pedagogy consisted of providing children with experiences related to their daily lives. Thus, learning is not dissociated from experience, so he understands that one learns by doing.

Regarding Mathematics, Dewey's thought contributed to theorizing about the number, so that, for Dewey, the number is not a property of objects that is manifested through the use of the senses, but is the result of a reasoning process arising from the empirical experience, so it is not intuitive, but an idea or a relationship formed by the measurement process (COSTA, 2014). Furthermore, it is worth mentioning that, from the Deweyan perspective of teaching, practical education based on learning through experience is central, therefore, the teaching of Mathematics is marked by the elaboration of projects with topics of interest to the student, which make it possible to put the student in action, which do not dissociate theory and practice and which also allow interdisciplinarity.

Decroly, on the other hand, is concerned with adapting any pedagogical attitude to the mentality and age of the child and, like Dewey, aims at group work based on projects related to the students' interests. In his theoretical perspective, he clarifies interest and curiosity. Interest is external, it is conscious desire;

curiosity is internal, it is a conscious or unconscious signal, but apparent to an external observer (DUBREUCQ, 2010).

According to Dubreucq (2010), the Decrolian method is intuitive and constructivist, so the child's internal personal resources are mobilized to develop their own knowledge. The children made their own school supplies using resources from workshops held at the school – carpentry, kitchen, laboratory, press, *etc.*, which allows the acquisition of autonomy and self-education.

According to the author, Decroly understands that one of the goals of the school is to assure the individual the chances of success in life. In this sense, the school, as it is, when classifying its students and assigning them diplomas, promotes only part of them, who, in turn, will have greater chances of success in life by presenting such documents.

In addition, he criticizes elements of the physical space and classical school organization: the corridor, aligned benches, the queues, the blackboard and the playground, which is not at all conducive to the discharge and exchange of energy by children, “For that then the school chose the 'bureaucrat's table' to deride the child of an 'employee culture' in which 'general culture is disqualified in favor of intellectual culture alone'?” (DUBREUCQ, 2010, p. 31). From his point of view, this type of school should disappear, as he understands that the natural environment – farm, fields, animals, for example – is ideal for stimulating children and that nature arouses their curiosity.

Other very important aspects of Decrolian pedagogical thinking are educational games and the global method. In his conception, the purpose of the game is to prepare for serious activity, in this sense, he builds, together with his team, several educational games that develop sensory, motor, visual, auditory and attention perceptions, for example (DUBREUCQ, 2010).

Its global method or globalization can currently be understood as interdisciplinarity. In this sense, school subjects are mobilized in an integrated way from a theme of the student's reality, and the contents are means for understanding the situation of the reality under study (VALENTE, 2019).

Therefore, its Pedagogy is globalizing and has structuring points called centers of interest, which represent ways in which the student fulfills himself and ways in which the individual behaves in society: feeding himself, fighting the weather, defending himself, acting and work. Through the centers of interest, activities are carried out in three stages: observing, associating and experimenting (VALENTE, 2019).

> Observation: it represents the lessons of things, the elementary lessons of natural sciences, geometry, calculus. 2) Association: in space and time, it replaces history and geography, conceived in addition to a broader point of view. 3) Expression: includes all language exercises, including spelling, color lessons, etc., as well as the so-called manual work and drawing (MOURA, 1931, p. 32-33 *apud* FERNANDES, 2019, p. 8).

Decroly was the inventor of the global method of reading and writing, also called the analytical or *Gestalt* method. Through this method, the student first learns the word and then the syllabic decomposition, that is, learning is “from the whole to the parts” (DUBREUCQ, 2010).

The global method also applies to the teaching of Mathematics when based on concrete to make comparisons: large, medium, inferior, superior, *etc*. Students' everyday situations are also used to solve real problems: comparing goods from a store, calculating the cost of a picnic and planning an excursion, for example (DUBREUCQ, 2010).

Fernandes (2018) analyzes the ideas contained in Decroly's work published in 1934, *El Calculo y La Medida en El Primer Grado de La Escuela Decroly*, and points out that the observation exercises, which can be individual or collective, they are considered as a starting point for problematizations and subsequent resolutions that allow the investigation of mathematical operations or mental resolutions. However, when the student cannot solve the exercise easily, then it is necessary to relate them “to the 'occasionally called' centers, which take advantage of the events that arise spontaneously, involving the exercises of observation, measurement, association and expression” (FERNANDES, 2018, p. 5).

In this way, such exercises value the problems of the student's life, also allow data to be collected by them and observation, considered as the basis of each center of interest, offers the possibility of measuring and comparing, as verified by the author in the analyzed work. Thus, she highlights that measurement – and there the fact of resorting to objects considered as natural units of measurement to make comparisons – and calculation are quite subject to observation. In this work, Decroly also discusses that different objects can be made available to the student in boxes or on a shelf for work with measures of capacity (cups, thimbles, cups, spoons, flasks, *etc.*) and even points to the use of games that involve mathematical operations, such as games with beans of different colors, movement games or games with rounds that allow children to play.

Regarding Kerschensteiner, popular education stands out, of which he is considered a pioneer, as his objective was to provide access to secondary education to the working population that did not have financial resources. His pedagogical thinking is also focused on the formation of human values, culture, citizenship, civics and national sentiment. The author is considered the creator of the School of Work, which aims to prepare students for a profession (RÖHRS, 2010b).

This type of school takes into account manual work, which goes beyond a technical skill:

> Manual work, if carried out properly, develops the faculty for logical thinking that can be applied to any other class of activity, and then deepens. There is a manual intelligence [...] that must be fostered at school, as it forms an integral part of each child's character (RÖHRS, 2010b, p. 25).

According to Röhrs (2010b), Kerschensteiner understands that manual work without intellectual effort becomes mechanical. Thus, the work is only considered in the pedagogical context if the intellectual effort - which can be understood as planning - comes before and is renewed during the execution of the work.

In this way, the School of Work conceived by him is based on an inseparable relationship between theory and practice, “the *pedagogical activity* that invariably provides material for *theoretical reflection* and dictates the way

forward" (RÖHRS, 2010b, p. 19, author's emphasis). This school intertwines the educational activity with the dispositions of each student in order to develop their inclinations and interests in the areas of work (LARROYO, 2010). Above all, in his conception, the purpose of all education is to clearly determine the student's will - what he wants - and, in the face of that, to show him a direction and the means for its realization (KERSCHENSTEINER, 2010).

Practical tasks or workshops in cooking, horticulture, mechanics, woodworking, animal handling, etc. are part of the curriculum and allow students to teachers teach content of school subjects, including Mathematics (RÖHRS, 2010b).

Ninow (2014) cites Mora (2003) to argue that Kerschensteiner, anchored in Dewey's ideas, was responsible for the assumptions and dissemination of project methodology in 20th century Germany. In addition, in the 1960s, work schools were created in Germany, under the ideology of project methodology.

The teaching of Mathematics in Kerschensteiner's perspective is very similar to that of Dewey. According to Almeida and Alves (2010), their teaching is given through workshops. Thus, a cooking class, for example, is not considered as a complementary class of the curriculum, but a class that allows the teaching of the subjects of the curriculum and that implies the teaching of a "Practical Mathematics" (ALMEIDA; ALVES, 2010, p. 42)

Freinet, like Kerschensteiner, has a socialist education in mind. Freinet lives in the context of the two world wars, becomes a leftist militant, fights for social equality and perceives a correlation between education and politics. It aims at the "people's school" for social transformation.

His educational thinking aims at the intellectual liberation of the working class, which is why he joins the Communist Party, visits schools in the Soviet Union and is influenced by Nadezhda Krupskaya, Lenin's wife. In 1933 the extreme right party grows, so the Freinets leave the public school and, with the support of the left, build their own school to serve poor Parisian children.

Freinet is against a verbalist school, so he articulates theory with praxis and understands that children and adults have the same nature. He values the surroundings of the classroom, nature. He understands that observations must be made in this environment, "Thus, the study of the environment remains the starting point, but what is essential, for Freinet, is not just observation. There is also, and above all, the need to understand and the need to act" (LEGRAND, 2010, p. 23).

> Freinet's pedagogy is based on four axes: cooperation (to build knowledge in a community), communication (to formalize, transmit and disseminate it), documentation, with the so-called book of life (to record daily of historical facts), and affectivity (as a link between people and between them and knowledge) (FERRARI, 2008, p. 7).

He is interested in the global method of reading and writing developed by Decroly, but uses it from his own perspective: "reading is inseparable from writing, but from writing composed of meaningful words and phrases, and not an abstract set of sounds" (LEGRAND), 2010, p. 19). To this end, he presents as a novelty the free text that refers to a school diary, the "book of life".

In addition, at their school, the children had binders called "work library" and there were different workshops that valued collective activity. According to

Scheffer (1995), these binders contained several subjects researched by the students and exercises to solidify the relationships already established during the classes. They can be considered as a type of study aimed at promoting the search for individual or group knowledge, because the pedagogical practice with these cards involved children's doubts and desires.

Freinet's Pedagogy is one of consensus, it is anti-hierarchical and presents cooperation as opposed to competition. His idea of work is in the sense of "putting the child into action" through the workshops.

> [...] the child wants to work. It is enough for him to enable the work-play[7] to which he aspires. For this, we will make available to you fields and gardens, animal husbandry, workshops, tools, essential machines and also manuals that will help you to overcome difficulties (FREINET, 2010, p. 124).

For Freinet (2010), education through work is not only a means of preparing man to produce social wealth, it goes beyond the utilitarian conception of human effort because it is, at the same time, motivation, stimulus and objective for the great and multiple life enterprise. Complementing, Scheffer (1995, p. 53) argues that education through work unites "intellectuality and manipulation, thought and action, concretization and thought". Freinet believes in Pedagogy as a way or even "the way" to transform humanity (LEGRAND, 2010). He calls his experience the Modern School, because he moves away from the New School.

In accordance with Freinet's thought, the learning of Mathematics undergoes transformations. He understands that "calculation must be an instrument of action on things" (LEGRAND, 2010, p. 17). Thus, Arithmetic is justified insofar as it serves to measure fields, weigh products, calculate prices, interest due or payable. "It is, therefore, about immersing school calculation in the surrounding life, converting it into a *living calculation*" (LEGRAND, 2010, p. 17, italic author's emphasis). In this way, Freinet understands that mathematical activities "come to life" when carried out based on the real needs of the day, through the resolution of practical problems.

> Freinet wanted to immerse mathematics teaching in concrete reality, using measurement as a privileged instrument. The teaching of measures, in turn, to take root, required concrete activities of manufacture, cultivation, livestock, marketing: soil extension, volume, weight, problems related to the amount of food for rabbits, chickens, purchase of seeds, sale of the harvest, all these occasions are useful for 'living calculation' (LEGRAND, 2010, p. 22).

According to the author, for Freinet it is not enough to teach the contents of school subjects, but it is necessary to satisfy his need to know in order to enrich his nature.

Scheffer (1995) argues that for Freinet, experimentation is considered the basis for teaching Mathematics. Furthermore, information brought by children, whether from their family or community, is valued for the elaboration of problem-situations, so that they will understand the "social bases of school

[7] Work-play refers to non-coercive work practices, but which lead children to enjoy work, as they are circumscribed in a learning context (FREINET, 2010).

arithmetic" (SCHEFFER, 1995, p. 54) and interpret their reality through these problematizations.

6. Final considerations

The educators presented in this article brought great contributions that caused changes in the educational field, starting with what refers to the pedagogical ideas of the 17th century, which, inspired by Rousseau, are linked to respect regarding the development of the student, as it considers their individualities and different age groups. Thus, they show how important it is to take into account the specificities of human nature, which contradicts, and much, the idea of "man-machine" (CAMBI, 1999, p. 330) that permeated the 17th and 18th centuries.

Other changes provoked by them are related to the conception of the child, of education, the role of the student and the teacher, the school and classroom model and with regard to the ways of thinking about Mathematics Education, including for the first childhood, through Maria Montessori.

Among these educators, when comparing some of their ideas, there are also similarities between them. By paying attention first to Herbart, a modern precursor of active methods, it is observed that he gives stand out in student self-government that, even being in an active position, does not refrain the teacher to direct and instruct him (DALBOSCO, 2018). Regarding the teaching of Mathematics, sensory experience is essential to arrive at the most elaborate mathematical ideas (KANG, 2012).

Among such changes, when comparing educators, similarities are found between them. Herbart, as a precursor of active methods (DALBOSCO, 2018), points to the centrality and individuality of the child, as well as other educators. Even in an active context that strives for student self-government, Herbatian pedagogy emphasizes the teacher's role to direct and instruct. Pestalozzi and Fröebel consider the child's developmental level and the teacher assumes the role of guide. From the perspective of these educators, the teaching of Mathematics also privileges the handling of concrete materials. In Pestalozzi, the global aspect is observed when certain activities involve words and the counting of their syllables, for example (NETA; GUTIERRE, 2020); on the other hand, in Fröebel, the use of language to structure what is observed in the real world stands out (HEILAND, 2010).

The child's freedom is so prioritized by Montessori that even the arrangement and size of furniture is adapted to the size of the child so that he can move around and become independent. Sensory perception is valued and this educator contributes to Mathematics Education by building different teaching materials.

Decroly, like Montessori, also builds educational games for teaching Mathematics and considers the global, that is, real situations that can be solved from an interdisciplinary perspective.

This perspective is also identified in Dewey, as teaching takes place through situations of the student's reality – Project Pedagogy. In Kerscheinsteiner and Freinet, the global character is also identified in the exploration of teaching workshops, which constitute a reason to awaken students to studies. For Kerscheinsteiner, the teaching of Mathematics takes place in manual practices, which can also be treated from an interdisciplinary aspect (RÖHRS, 2010) and

which are similar to the Deweyan perspective (NINOW, 2014). Kerscheinsteiner follows Freinet's Cooperative Pedagogy, in which students learn in workshops, that is, from the real, with a view to applying what they learn in practical life.

Therefore, when considering the educators presented in this paper, traces of a school Mathematics Education that considers the student's experience with the empirical world are evidenced. The teacher develops his practice from objects or situations in the student's reality - very simple and everyday objects, educational toys, problem situations, projects or workshops – so that, using sensory perception and/or language, under an interdisciplinary approach, one can arrive at the most elaborate ideas of mathematical concepts or contents.

References

ADORNO, Thais Lira França; MIGUEL, Maria Elisabeth Blanck. A metodologia de Pestalozzi e o ideário da Escola Nova. **Acta Scientiarum**, v. 42, publicação contínua, p. 1-11, 2020.

ALMEIDA, Danilo Di Manno de; ALVES, Maria Leila. A recepção da “escola do trabalho” no contexto brasileiro. In: RÖHRS, Hermann. **Georg Kercheinsteiner.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, p. 39-52. Coleção Educadores.

ANDRADE, Guilherme Trópia Barreto de. Percursos históricos de ensinar ciências através de atividades investigativas. **Revista Ensaio Pesquisa em Educação em Ciências,** Belo Horizonte, v. 13, n. 01, jan./abr., p. 121-137, 2011.

ARANHA, Maria Lúcia de Arruda. **História da Educação e da Pedagogia:** geral e do Brasil. São Paulo: Moderna, 2012, 685 p.

BOTELHO, Louise Lira Roedel; CUNHA, Cristiano Castro de Almeida; MACEDO, Marcelo. O método da revisão integrativa nos estudos educacionais. **Gestão e Sociedade**, v. 5, n. 11, maio/ago., p. 121-136, 2011.

CAMBI, Franco. **História da Pedagogia.** Tradução de Álvaro Lorencini. São Paulo: Fundação Editora da UNESP (FEU), 1999, 701 p.

COSTA, David Antonio da. The conceptions and contributions of Pestalozzi, Grube, Parker and Dewey for teaching Arithmetic in Elementary Level: the concept of number. **História da Educação**, Porto Alegre, v. 18, n. 42, jan./abr., p. 37-59, 2014.

DALBOSCO, Claudio Almir. Uma leitura não-tradicional de Johann Friedrich Herbart: autogoverno pedagógico e posição ativa do educando. **Educação e Pesquisa**, São Paulo, v. 44, p. 1-18, 2018.

DESCARTES, René. **Discurso sobre o método.** São Paulo: Abril, 1984. Coleção Os Pensadores.

DEWEY, John. Vida e educação. *In:* WESTBROOK, Robert Brett. **John Dewey.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, p. 69-109. Coleção Educadores.

DUBREUCQ, Francine. **Jean-Ovide Decroly.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, 156p. Coleção Educadores.

FERNANDES, Juliana Chiarini Balbino. A matemática na proposta de Decroly: análise do livro *"El Calculo y La Medida em El Primer Grado de La Escuela Decroly*". *In:* IV Encontro Nacional de Pesquisa em História da Educação Matemática, 4º, 2018, Campo Grande. **Anais...** Campo Grande: Universidade Federal do Mato Grosso do Sul, 2018, p. 1-15.

FERNANDES, Juliana Chiarini Balbino. Centros de Interesse no ensino primário: há saberes aritméticos? In: XVII Seminário Temático Materiais Didáticos e História da Educação Matemática, 17º, 2019, Aracaju. **Anais...** Aracaju: Universidade Federal de Sergipe, 2019, p. 1-12.

FERRARI, Márcio. Célestin Freinet: o mestre do trabalho e do bom senso. Nova Escola, São Paulo, jul. 2008. Edição especial grandes pensadores. *In:* **Elaboração de Projetos:** eixo 1. Universidade Federal do Ceará, 2008. Retrieved from: http://www.virtual.ufc.br/cursouca/modulo_4_projetos/conteudo/unidade_1/Eixo1-Texto7.pdf Access in: Jan 4th, 2021.

FERREIRA, Jefferson dos Santos; SANTOS, Ivanete Batista dos. Apropriações do Método Intuitivo de Pestalozzi em Propostas Para o Ensino de Saberes Elementares Matemáticos em Revistas Pedagógicas (1890-1940). *In:* **JIEEM**, v. 11, n. 1, p. 13-26, 2018.

FREINET, Célestin. A educação do trabalho. In: LEGRAND, Louis. **Célestin Freinet.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, p. 67-139. Coleção Educadores.

FROEBEL, Friedrich Wilhelm August. A educação do homem. *In:* HEILAND, Helmut. **Friedrich Froebel.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010. p. 41-117. Coleção Educadores.

GASPARIN, João Luis. As ideias de Pestalozzi no Brasil. *In:* SOËTARD, Michel. **Johann Heinrich Pestalozzi**. Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, p. 31-40. Coleção Educadores.

GAUTHIER, Clermont. Da pedagogia tradicional à pedagogia nova. *In:* GAUTHIER, Clermont; TARDIF, Maurice. (Org.). **A Pedagogia:** teorias e práticas da Antiguidade aos nossos dias. Petrópolis: Vozes, 2010, p. 175-202.

HEILAND, Helmut. **Friedrich Froebel.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, 138p. Coleção Educadores.

HILGENHEGER, Norbert. **Johann Herbart.** Recife: Fundação Joaquim Nabuco, Editora Massangana, 2010, 148 p. Coleção Educadores.

KANG, Hyun Young. A study of the mathematics education of Herbart. **The Korean Journal for History of Mathematics,** Yuseong-gu, v. 25, n. 2, p. 35-55, 2012.

KERSCHENSTEINER, Georg. Alma do educador e o problema da formação do professor. In: RÖHRS, Hermann. **Georg Kerschensteiner.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, p. 84-123. Coleção Educadores.

LARROYO, Francisco. História geral da pedagogia. In: RÖHRS, Hermann. **Georg Kerschensteiner.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010. Coleção Educadores.

LEGRAND, Louis. **Célestin Freinet.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, 150 p. Coleção Educadores.

LOURENÇO FILHO, Manoel Bergstrom. Introdução ao estudo da nova escola: bases, sistemas e diretrizes da pedagogia contemporânea. *In:* HILGENHEGER, Norbert. **Johann Herbart**. Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010. Coleção Educadores.

MARTINEAU, Stéphane. Jean-Jacques Rousseau: o Copérnico da pedagogia. In: GAUTHIER, Clermont; TARDIF, Maurice. (Org.). **A Pedagogia:** teorias e práticas da Antiguidade aos nossos dias. Petrópolis: Vozes, 2010, p. 149-171.

MONTESSORI, Maria. Pedagogia científica: a descoberta da criança. *In:* RÖHRS, Hermann. **Maria Montessori.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010a, p. 51-93. Coleção Educadores.

MONTESSORI, Maria. A criança. *In:* RÖHRS, Hermann. **Maria Montessori.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010b, p. 93-131. Coleção Educadores.

NEITZEL, Odair. Experiência educacional em Dewey e Herbart. **Theoria** - **Revista Eletrônica de Filosofia,** Pouso Alegre, v. VII, n. 18, p. 120-142, 2015.

NETA, Olivia Morais Medeiros; GUTIERRE; Liliane dos Santos. O ensino de Matemática no pensamento de Comênius, Pestalozzi e Montessori. **Educar em Revista**, Curitiba, v. 36, p. 1-22, 2020.

NINOW, Valmir. **Projetos de trabalho no Ensino Médio:** uma possibilidade para o ensino e aprendizagem da matemática. 2014. 158f. Dissertação (Mestrado em Ensino de Ciências e Matemática) – Programa de Pós-Graduação em Ensino de Ciências e Matemática, Universidade Luterana do Brasil, Canoas, 2014.

OLIVEIRA, Marcus Aldenisson de. *Pedagogia Intuitiva* da Escola Elementar de Pestalozzi: como se ensinava Aritmética? **Bolema [on-line],** Rio Claro, v. 31, n. 59, p. 1005-1031, 2017.

PESTALOZZI, Johann. El método. In: SOËTARD, Michel. **Johann Pestalozzi.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010. Coleção Educadores.

REINHOLD, Simone; DOWNTON, Ann; LIVY, Sharyn. Revisiting Friedrich Froebel and His Gifts for Kindergarten: What Are the Benefits for Primary Mathematics Education? *In:* Annual Meeting of the Mathematics Education Research Group of Australasia (MERGA), 40th, 2017, Melbourne. **Paper...** Melbourne: ERIC, 2017, p. 434-441.

RÖHRS, Hermann. **Maria Montessori.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010a, 142p. Coleção Educadores.

RÖHRS, Hermann. **Georg Kerschensteiner.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010b, 142p. Coleção Educadores.

SAVIANI, Dermeval. Pedagogia: o espaço da educação na universidade. **Cadernos de Pesquisa,** São Paulo, v. 37, n. 130, jan./abr., p. 99-134, 2007.

SCHEFFER, Nilce Fátima. **O encontro da Educação Matemática com a Pedagogia de Freinet.** 158f. Dissertação (Mestrado em Educação Matemática) – Universidade Estadual Paulista Júlio de Mesquita Filho, Rio Claro, 1995.

SOËTARD, Michel. **Johann Pestalozzi.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, 112p. Coleção Educadores.

SILVA, Circe Mary Silva da; SOARES, Waléria de Jesus Barbosa. Ideias Pedagógicas de Montessori no Brasil: Contributos à Educação Matemática**. REMATEC: Revista de Matemática,** Ensino e Cultura, Belém, ano 15, fluxo contínuo, p. 195-211, 2020.

VALENTE, Wagner Rodrigues. Aritmética e interdisciplinaridade: ecos da história da educação matemática. **Educar em Revista**, Curitiba, v. 35, n. 75, p. 305-318, maio/jun., 2019.

WESTBROOK, Robert Brett. **John Dewey.** Recife: Fundação Joaquim Nabuco/Editora Massangana, 2010, 136p. Coleção Educadores.

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