Artigo

The negative number in Davydov's teaching proposition: necessities for its introduction

Lucas Sid Moneretto Búrigo[1], **Ademir Damazio**[2]

Universidade do Extremo Sul Catarinense, UNESC, Criciúma-SC, Brazil

Abstract

The difficulties presented by Brazilian students in the appropriation of the negative number reported in surveys contributed to this article preparation. Concerned about these difficulties, we turned to Davydov's proposition, which is the subject of this study. The research problem is: what are the needs created in the particular tasks of Davydov's educational organization for the introduction of the negative numbers concept, in school situation? It is a qualitative research method in the literature whose source analysis are four tasks presented in the student textbook belonging to Davydov's proposition. The review process was developed through necessity category: conceptual and pedagogical. The research shows that the need to study the negative numbers is based on the fact that only they enable the resolution of some cases of subtraction and equation. To provide the meaning of opposite to the negative number, the main need is the change from scalar quantities to the vector one.

Keywords: Proposition davydovian. Negative numbers. Conceptual necessities.

Resumo

As dificuldades apresentadas por estudantes brasileiros na apropriação do número negativo relatadas em pesquisas contribuíram para a elaboração desse artigo. Preocupados com tais dificuldades, recorremos à proposição davydoviana que é objeto desse estudo. O problema de pesquisa é: quais são as necessidades criadas nas tarefas particulares da organização de ensino davydoviano para a introdução do conceito de número negativo, em situação escolar? Trata-se de uma pesquisa qualitativa na modalidade bibliográfica cuja fonte de análise são quatro tarefas presentes no livro didático do estudante pertencente à proposição davydoviana. O processo de análise se desenvolveu por meio da categoria necessidade: de ordem conceitual e pedagógica. A pesquisa evidencia que a necessidade do estudo dos números negativos decorre de que apenas eles possibilitam a resolução de alguns casos de subtração e equação. Para propiciar o significado de oposto ao número negativo, a necessidade principal é a passagem das grandezas escalares para a vetorial.

Palavras-chave: Proposição davydoviana. Números negatives. Necessidades conceituais.

Introduction

Regarding the teaching of mathematics in the Brazilian educational context, many concepts are pointed out as difficulties indicators by the students. Among them, the concept of negative number has originated studies on its learning and teaching. The research of Amorim (2012) is an indicator of the state of the art revealed in research that deals with this issue from the period 2001 to 2010. Silva (2012) identified

1 Master in Education at then Universidade do Extremo Sul Catarinense (UNESC). Professor at the University of extreme southern Santa Catarina E-mail: lucassid@hotmail.com

2 Phd in Education Universidade Federal de Santa Catarina (UFSC) Professor in the Graduate Program in Education of Universidade do Extremo Sul Catarinense (UNESC). E-mail: add@unesc.net

the elaborations of students when developing guiding activities of teaching with the whole numbers, more specifically with the negative ones. Damazio (2001) investigated the teaching process in school environment, with attention to the tasks organization of the study by a professor, who articulated situations of the daily life of empirical basis with the theoretical content of the concept of relative integer. These three studies, among so many feasible of indications, are informative for the problematization and relevance of research in focus for two reasons. One of them concerns the evidences of the complexity of this concept in the school context. The other for pointing that even the research with innovative proposals exhibit identical content to those proposals with which they express disagreements, once that they bring in essence a conceptual empirical basis and, consequently, develop the same type of thoughts in the students. It is realized such inconsistency by bringing in essence a conceptual empirical basis, that consequently, develops the same type of thought in students. As Davýdov says (1982), there is a change in the method of teaching, but the content of the concept remains the same.

If the academic and scientific investigations show a pedagogical panorama that does not favor the conceptual appropriations of the mathematical concept in reference, the same concern is expressed in the document that serves as a guideline for the organization of education in Brazil, the National Curriculum Parameters (PCNs), when he says: "[...] in school, the study of the whole numbers is usually surrounded by difficulties and the results, as regards their lifelong learning of Fundamental Education, has been quite unsatisfactory." (BRAZIL, 1998, p. 97).

The weaknesses indicated, up to the moment, refer to the study of other educational possibilities. That is why, in the present study, the centrality is the teaching that contrasts to the insistence on empirical content to the concept of negative number, which remains even in those proposals which say they are innovative as explained by Amorim (2012), Silva (2012) and Damazio (2001). For that, the reference is the Davýdov's teaching proposition and collaborators for the Mathematics subject, resulting from the research developed for more than twenty-five years. Davídov (1988) showed to be prone to the organization of teaching, with views to the psychological development of the students, unlike that one which was achieved by the education systems in Russia. His assumption is that, since the transition from pre-school to school, education offers the appropriation of scientific concepts by students, so that they can develop in conditions to understand the world with theoretical basis, instead of empirical.

Davýdov (1982) believes that the student, when joining the school, find a teaching organization completely distinct - both in method and content - from that he or she will live in pre-school or in daily learning situation. His justification for this assumption is that, when ingressing to school, the student falls into a new place in the social context, which will mark his or her development promoted by a new main activity: the study activity that, gradually, overlaps over the game which until then predominated. Davídov and Slobódchikov (1991) consider that the student only develops the studying activity, through the assimilation of scientific concepts when he or she experiences an inner necessity and a motivation for such. The needs and the motives are those that guide the students in the process of knowledge appropriation, which generates the human development at the theoretical thought level. The organization mode of education proposed by Davídov (1988) is composed of 1) 'studies tasks', which are met by 2) six 'study actions', each of them was developed by a set of 3) 'private tasks', which in turn require procedures or 'operations'. For Davídov and Slobódchikov (1991), the study tasks allow the analysis, by the students, of origin conditions of knowledge, as well as the domain of their generating procedures. Davídov (1988, p. 209) defines a system of studies tasks related to Mathematics for the Fundamental teaching, which is:

1) introduction of students in the sphere of relationships between quantities: formation of the abstract concept of mathematics quantity;
2) demonstration, for children, the multiple ratio of the quantities as multiple general shape of the number: formation of the abstract concept of number and understanding of fundamental inter-relation between its components (the number is derived from multiple relations of quantities);
3) Successive introduction of students in the area of the different particular types of numbers (natural, float, negative): training of concepts about these numbers as a manifestation of multiple relation general of quantities in certain concrete conditions;
4) demonstration to students of the univocal character of the structure of the mathematical operation (if you know the value of the elements of the operation it may be determined unambiguously the value of the third element): formation of the understanding on the inter-relation of the elements of fundamental arithmetic actions.

For each of these tasks, the following actions of study highlight:

1) transformation of the task data in order to reveal the universal relation of the object of study;
2) modeling of differentiated relation in the form of object, graphic or through letters;
3) Transformation of the relation model to study their properties in 'pure';
4) the construction of particular tasks system to resolve by a general procedure;
5) control on the compliance of the previous actions;
6) evaluation of the general procedure assimilation as a result of the solution of the given task of study. (DAVÍDOV, 1988, page 181).

In the present study, the focus is to private tasks relevant to the first two actions related to the concept of negative number present in the textbook of -the sixth school year (ГОРБОВ et al., 2007). The assumption is that in them it is revealed the needs of conceptual and pedagogical order that guide and motivate the students to the development of the activity of the study when, within the school context, the basis of study and learning is the aforementioned mathematical concept.

It is in this context that emerges from the research problem: What are the needs created in particular tasks of the organization of davydovian teaching for the introduction of the concept of negative number, in school situation? This leads to the objective to identify and analyze the needs and their characteristics of conceptual and pedagogical order that move the determination and preparation of peculiar tasks relating to the teaching of the concept of a negative number. In addition, it requires attention to the way that the relation among the quantities - essential genetic basis of the theoretical concept of number - is presented in numerical singularity, negative, and manifested in particular tasks developed by the students.

The assumption that moves the analysis of the object of study is that, at the davydodian mode of teaching organization, the private tasks - reference of analysis of this study - are inserted into needs contexts. One of them regards the conceptual limitation of the numbers until then studied, the positive ones, in previous years (until the fifth year). The other corresponds to the process of introduction of negative numbers in the sixth year. A necessity, according to Leontiev (1978), is related with the lack of something indispensable for something. That is: "when the organism lacks of certain elements essential for life, this is manifested in that it requires these elements or, in other words, complains the satisfaction of their needs" (LEONTIEV, 1978, p. 341). When approaching the concepts of activity and psyche, based on Leontyev, Davídov (1988, p. 254) understands that "[...] activity has its peculiar premise: the need as a shortage state and of stimulation of the organism [...]".

The needs are, therefore, the basis of analysis of the particular tasks that refer to the conceptual development of negative number. They were comprised with the following ideas: a shortage state (DAVÍDOV, 1988) and lack of something (LEONTIEV, 1978).

In the process of analysis with the peculiarity of a qualitative research, the reference were private tasks introductory of the concept of negative number, extracted from the sixth year textbook (ГОРБОВ et al., 2007) of davydovian teaching preposition. In addition, the professor´s guide book has been adopted (ГОРБОВ et al., 2006), because it contains details of the developing concept and its contents, as well as didactic procedures of each one of the particular tasks to be performed by the students.

Among the several private tasks, it was selected for the analysis those that reflect the needs for the didactic process of introduction of the concept of negative number in the sixth school year. This has requested the dialog with studies related to mathematical principles (CARAÇA, 2010; WINTERLE, 2000) and, also, investigations regarding the davydovian proposition (Rosa, 2012; Souza, 2013), for subsidizing the identification and the needs understanding, respectively, of mathematical and pedagogical conceptual characteristics, as well as the articulation between both.

The organization of the analysis process follows the following procedure: 1) full transcript of the formulation of the task, extracted from the textbook (ГОРБОВ et. al., 2007); 2) the resolution process of the tasks in dialog with two categories: the conceptual basis of negative number and pedagogical basis. The first concerns the ideas that characterize the essentiality of the concept of number in its singularity, the negative. The second, of pedagogical basis, relates to the peculiarities of the process of teaching organization of the mathematical concept in focus. In other words, what leads to the preparation and execution of a new special task to which the student appropriates of the multiple inter-relations and of the meanings of the said mathematical concept.

It is in this context that, hereinafter, shall be evidenced some particular tasks for the teaching of the negative number, with attention to the needs that they provide to their preparation linked to conceptual development.

Negative numbers in the Davýdov's teaching proposition and collaborators

We will analyze the first tasks that bring the student close to the concept of a negative number. Basically, they make the student understand the limitations of numbers until then studied, the positive ones, because they do not allow to express the result of a peculiarity of arithmetic operation of subtraction, as well as the solution to a specific case of the equation. The first task in particular (ГОРБОВ et al., 2007, p. 46) puts the student in situation of need to extrapolate their knowledge of positive numbers and start their transit through the negative ones. In this sense, it brings an element that is reference in all concepts, since the first year: the number line. In it, the central idea is the route in the opposite direction to the positive numbers. This is not a simple movement of displacement, but mediated by subtraction operation, that is, the - b, in that first the b is shorter than the a, but, finally, turns out to be greater.

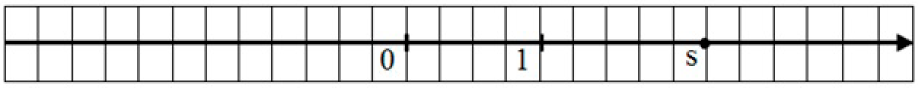
Task 1:

In the number line, according to figure 1, identify the number corresponding to s.

(a) Mark on the line number the site corresponding to the numbers that represent: s - 1; s - 2; s - 3; s - 4.

b) Write down the marked numbers (ГОРБОВ et al., 2007).

Figure 1: Successive subtraction: need generator of negative numbers



Source: Adaptation from Горбов et al. (2007)

It is observed in figure 1, that the measurement unit is composed of four meshes, because between the number zero and 1 there are four parts. This leads to the anticipation that positive and negative numbers will present in the field of the real numbers - with the idea of measure - and not only regarding to the whole numbers. It is noted, still, also, that each number in the line number corresponds to four fourths ($\frac{4}{4}$), for example: 1 = $\frac{4}{4}$ In the same way, the number two consists of eight forths ($\frac{8}{4}$), three of twelve ($\frac{12}{4}$) and so on.

Under this understanding, to seek the number *s*, it is observed that it is between the number two and three. With the aid of the parts of the mesh, it is observed that it is the number nine forths ($\frac{9}{4}$). The number identification s will assist in the resolution of the question b.

The situation *an* of task 1, with the aid of the line number, has as objective to reach the part previous to zero in the line mentioned for negative numbers. For this reason, it is resorted to the subtraction operation to fulfill this purpose.

In other words, the record of subtraction from one to four units of *s* makes you scroll down the path in accordance with the direction: from the positive numbers to the negative ones. That is, it lies to the students the situation of ignoring the numbers that are known, the positive ones, and reach the unknown, that is the negative ones. It is worth highlighting that, until then, the davydovian proposition, had not still proposed the exceeding of the number zero in the number line.

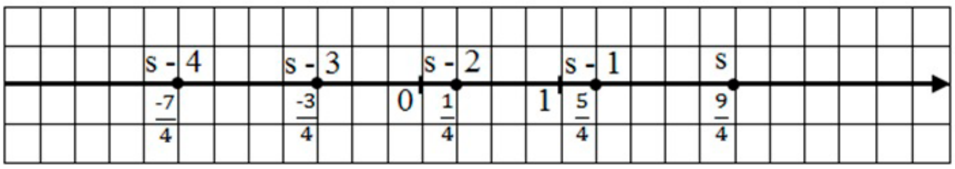
Due to the fact that in the context of the concept of number missing the study of negative in the davydovian proposition, it is verified that the situation *a* creates the need for conceptual appropriation that there are numbers that are located before the zero, negative ones.

In the context of the creation of the aforementioned conceptual need, it is observed that this need is bound to the question of pedagogical preparation of relevant task with the purpose of students to perceive the possibility of another characteristic of the number: to be negative or to be positive that articulates with the elimination of restrictions on the subtraction (it was possible only if the subtrahend was smaller than the minuend) and the zero as reference for relativity (being a positive or negative number).

In continuation to the situation *a*, the next question has the purpose that be recorded in the numeric line the numbers corresponding to the subtractions: s - 1; s - 2; s - 3; s - 4. In this circumstance, the subtraction becomes a mediator concept for the emergence of negative numbers. Caraça (2010, p. 92), when analyzing the subtraction between two real numbers an and *b*, considered the difference of *a - b* as a "[...] relative number, what shall we say positive, negative or zero, as it is $a > b$, $a = b$, $a < b$.".

Although the study context turns to negative number, the possibility of its emergence to be successive subtraction is the same that is presented - in the davydovian proposition - in the first school year for the introduction of the number zero, as it is indicated in the studies by Rosa (2012) and Souza (2013). However, we emphasize that there is no difference in both processes. This is because, in the first academic year, the successive subtraction occurs in the context of the singularity numeric of the natural numbers type: 4 - 0 = 4, 4 - 1 = 3, 4 - 2 = 2, 4 - 3 = 1 and 4 - 4 = 0. In its turn, in the sixth year, the subtraction refers, besides the extrapolation to previous part to zero, the numerical singularity of the rational numbers, as, for example, $\frac{9}{4} - 3 = -\frac{3}{4}$.

Figure 2: Representation of the corresponding numbers to the successive subtractions from *s*



Source: Resolution of the situations based on the orientation of Горбов et al. (2007)

This task, when proposing subtraction, especially in cases in which the minuend is smaller than the subtrahend, has an epistemological purpose articulated with the pedagogical one, because it puts the student facing an unknown and sharp situation of a possibility for a concept not appropriate yet.

This means to say that the obtaining of the last two results (*s - 3* and *s - 4*) lacks conceptual signification and creates the need for a task that makes explicit the intervention of the teacher. In conceptual terms, the need generated in the resolution of part of that task is that something is missing: the negative number (LEONTIEV, 1978).

This is because of the absence of an essential element for the resolution of the subtraction operation when the minuend is smaller than the subtrahend. In other words, such absence demands the satisfaction of the need to study a new mathematical concept in the numerical field.

As regards the subtraction and the number line, we inform that such concepts are introduced in the davydovian proposition in the first school year (Rosa, 2012; Souza, 2013). In this way, we see, through the task 1, which the subtraction of a – *b* is*,* with *b* greater than *a*, performed with the aid of the number line, which elicits the need for the study of a new mathematical concept, the negative number.

Thus, Davýdov and collaborators fall back on to a concept already covered in their proposition of teaching to generate the need to study a new concept. This means that the referred teaching organization aims that students reach a new conceptual development. But for this to occur, it will use the concepts that were appropriated by students and that allow to study new concepts.

The task 2, as we will see below, has the same purpose of the first, however with it we resort to another concept which also will generate the need to study the negative number.

Task 2:

Select the equation that can be solved and find its solution (ГОРБОВ et al., 2007):

a) x + 178 = 356;
b) x + 356 = 178;
c) x · 178 = 356;
d) x · 356 = 178.

The four situations that make up the present job have a content of evaluation in relation to the ability of the students to highlight, among the equations, those whose resolution occurs by known methods by them, as also to reach appropriate solutions (ГОРБОВ et al., 2006).

In situations *a, c* and *d,* the number to be assigned to the unknown factor x to achieve equality between the two members of the equations has a characteristic to be positive. However, in *b,* any positive number promotes the inequality between both members of the equation. This allows us to state that this number will not be the solution of the equation of the situation *b*, because it only promotes the inequality between both members of the equation.

For such inequality, the situation *b* generates the need of a number that is distinct from the positive, i.e. the negative, since only it has the characteristic of being added to a positive number and the result of the sum to be smaller than the positive number to which was added.

Again we have detected the need of a conceptual number with a characteristic not studied yet in the davydovian proposition, i.e. the negative, since only it allows the equation x + 356 = 178 to have as a solution: x = - 178.

As regards the result of the equation *b*, Горбов et al. (2006) emphasize that it has only a formal character (procedural) and does not indicate that the student still has some conceptual notion of negative numbers. The authors also do not discard the possibility of obtaining the erroneous dimensions of the result, for example, *x* = 178. This shows that the attention of the student does not turn to the understanding of the essence of the task; he or she only performs it formally, using procedures that he or she dominates.

The understanding of the meaning of the equation of the question b, together with the search for a way to resolve it, is something emerging and one of the immediate requirements, because it needs a concept, not studied yet in the teaching proposition under analysis.

From the moment that was created the need for the study of negative number through appropriate concepts in previous years (subtraction and equation), it is now time to satisfy that need. Thus, the next two particular tasks will be directed to the introduction of a negative number.

To fulfill the purpose of the introduction of negative numbers, the third particular task presents two important characteristics. One of them is that it promotes the transfer of the method used in the measurement of the scalar values (length, area, volume, mass, etc.) for the measurement of vector quantities that are represented by vectors. The other establishes that, when measuring one vector, it is a condition that another is taken as the unit.

When it comes to the method used for the measurement of quantities, it is worth highlighting that for Caraça (2010) the measurement consists in the comparison of two quantities of the same species: two lengths, two weights, two volumes, among others. In this process, it is outlined three phases linked together: the choice of the quantity taken as a unit; the comparison of the unit with the quantity to be measured; and the expression of the result of this comparison by a number.

In a similar way, Costa (1866) considers that measuring a quantity corresponds to the determination of how many times it contains another quantity of the same species taken as a unit. There is, then, a relation of the numbers with their quantities, i.e.: "[...] the numbers are expressions of measurement of quantities." (COSTA, 1866, page 9).

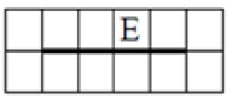
It is in this context of quantities measurement that is started the study of particular tasks concerning the introduction of negative numbers.

Task 3:

a) from the unit E (figure 3), build a length segment

1) D = E + E; 2) B = E + E + E; 3) C = E + E + E + E.

Figure 3: Length



Source: Adaptation from Горбов et al. (2007).

b) Measure the length and, D, B and T where T = $\underbrace{E + E + \cdots + E}_{n}$, with the unit E:

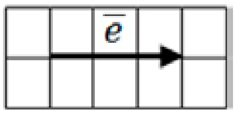
$\frac{E}{E} =$; $\frac{D}{E} =$; $\frac{B}{E} =$; $\frac{T}{E} =$.

c) Would there be another way to record the results of the measurement of these quantities?

d) build the vectors:

$$1)\ \bar{d} = \bar{e} + \bar{e}\ ;\ 2)\ \bar{b} = \bar{e} + \bar{e} + \bar{e}\ ;\ 3)\ \bar{c} = \bar{e} + \bar{e} + \bar{e} + \bar{e}\ ;$$

Figure 4: Vector



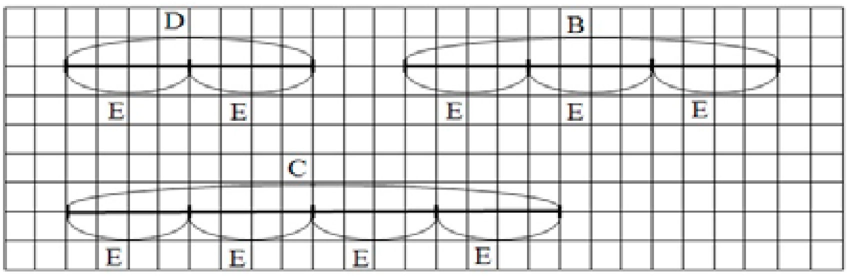
Source: Adaptation from Горбов et al. (2007).

e) Be $\bar{t} = \frac{\bar{e}+\bar{e}+\cdots+\bar{e}}{n}$, how to specify the relation of the vectors $\bar{d}$, $\bar{b}$, $\bar{e}$ and $\bar{t}$ with the vector $\bar{e}$ (connected by its module and sense with the module and the sense of the vector $\bar{e}$? (ГОРБОВ et al., 2007).

We understand that the situation *a*, *b* and *c* have the specificity of reproducing the appropriate knowledge by students started in the first school year, since the study of the concept of number in the teaching organization in the analysis of the introduction of the quantities (length, area, volume, mass, etc.), because the number is based on the relation between them (Rosa, 2012).

The situation *a* s is characterized by requiring the construction of segments, as provided for in items 1, 2 and 3, through the unit E. Thus, segments D, B and C have equal lengths, respectively, to two, three and four segments of E (figure 5).

Figure 5: Representation of the segments D, B and C



Source: Resolution of the situations based on the orientation of Горбов et al. (2007).

In the situations *b* and *c,* occur the specification of the relations among the segments and, D, B and T with unit E. Each relation presupposes the comparison between two lengths. For Caraça (2010), the comparison between two quantities of the same species means to measure one of the quantities greatness with the other taken as the unit, in which is done the verification of how many times the unit fits in the quantity to be measured and such amount of times is expressed by a number.

In the context of the comparison of lengths that results in a number, we emphasize that the davydovian proposition for the first school year, in accordance with Rosa (2012), two formulas are adopted that portray the theoretical concept of number: $\frac{A}{B}$ = c And A = cB (A represents the quantity to be measured, B the quantity taken as the unit and *c* he number that expresses the relation between the two quantities).

Both formulas represent two different relationships between quantities of the same species: the divisibility and multiplicity. In the first relation, A, when divided into equal parts to B, it will result in *c* parts. While in the second, A will be equal to *c* times B (ROSA, 2012).

The two relations mentioned are present in situations *b* and *c*. In *b,* it is resorted to the relation of divisibility among, E, D, B and T with the unit E. This relation indicates how many parts, equal to the quantity E, it is possible to divide E, D, B and T. after proper comparison, the respective relations are expressed in the following way; $\frac{E}{E} = 1; \frac{D}{E} = 2; \frac{B}{E} = 3; \frac{T}{E} = n.$

In situation *c,* the process of comparison among the quantities E, D, B and T, taken E as a unit, expresses the relationship of multiplicity, i.e. E = 1E, D = 2 and B = 3E and T = nE.

It is observed that from the situation *a* up to *c*, the relations are limited to the quantities of length. In this sense, we inform that the length was constituted as a scalar quantity and to determine the number that, until then, in the Davýdov and collaborators´ teaching organization is only positive, it is enough to check how many times the quantities taken as the unit of measure fit in the quantity to be measured.

The situation *d,* when requesting the construction of vectors[3], allows the observation of the use of a different quantity from previous situations, that is, the vectorial. As regarding the vectorial quantities, Winterle (2000, p. 1, author's writings considers that they

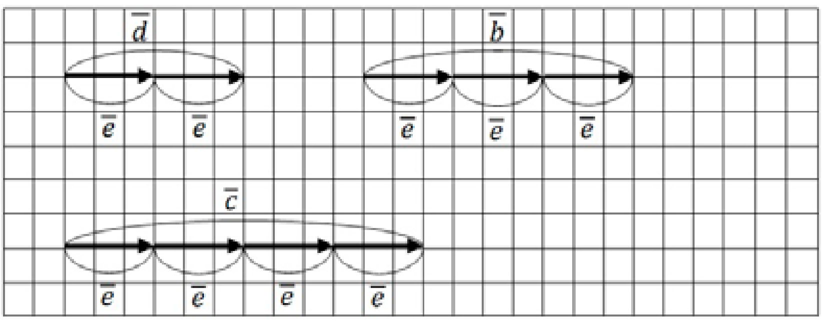
> [...] are not completely defined only by their module, i.e. by the number with their corresponding unit. We talk about the vectorial quantities, that to be perfectly characterized we need to know their *module* (or length or intensity), their *direction* and their *sense*.

In other words, the use of the vectorial quantity - generically represented through a vector - lacks the analysis of other characteristics that are not presented with the scalar quantity, i.e. the sense and direction.

This allows us to state that the vectorial quantity also carries the length, but, at the same time, it is not limited to it, because it contains two other characteristics: the sense and direction.

In the context of vectorial quantity, the situation *d*, unlike the situation *a*, requires that are constructed the vectors $\bar{d}$, $\bar{b}$ e $\bar{c}$ (figure 6) of same sense and direction of , instead of just segments, as it was seen in the situation *a*.

Figure 6: Representation of vectors $\bar{d}$, $\bar{b}$ and $\bar{c}$ through $\bar{e}$



Source: Resolution of the situations based on the orientation of Горбов et al. (2007)

3 In the previous chapter to the negative number in the Professor´s guide books and the didactic of the student occurs the introduction of the vector.

The situation *e* has the specificity of enabling the beginning of comparison between two vectors. In other words, it is performed the measurement of the vectors that, as it had occurred with scalar quantities - since the first school year - will also be represented by a number. However, the comparison of two vectors is not limited only to their module - which does not occur when comparing two lengths - it also lacks the comparison of the sense and direction of both of them.

Regarding to what corresponds to the comparison of the sense of the vectors, we affirm that this is only possible because they have the same direction, i.e. due to the vector have a direction and, furthermore, due to it make part of $\bar{b}$, $\bar{d}$ e $\bar{t}$ it means that it has the same direction as $\bar{e}$.

Due to the vectors $\bar{t}$, $\bar{d}$, $\bar{b}$ and $\bar{e}$ have the same sense of unit $\bar{e}$ means that the comparison between them is represented by the same numbers of situations *b* and *c*, the positive ones. Therefore, the result of this comparison is: $\bar{t} = n\bar{e}$; $\bar{d} = 2\bar{e}$; $\bar{b} = 3\bar{e}$; $\bar{e} = 1\bar{e}$

This situation under review is characterized for generating the need for comparison of the senses of the vectors, in addition to the module, to be able to specify the relationship between the two vectors. Such need for comparison of the sense of the vectors stems from that in the Davýdov and collaborators' teaching organization occurs the transition from the use of scalar quantities to the Scalar Vector, represented by vectors.

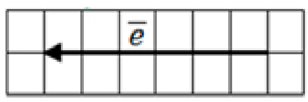
However, we emphasize that the task 3 has not enabled yet the introduction of negative number. This allows us to note that there is still something different missing, which at the same time, it is essential for the study of such a number. With respect to such failure, we inform that the task 4 will close the gap that still exists.

Task 4:

a) build the vectors:

$$1)\ \bar{d} = \frac{2}{3}\bar{e}\ ;\ 2)\ \bar{d} = \frac{2}{3}\bar{e}\ ;\ 2)\ \bar{m} = 1\frac{5}{6}\bar{e}\ ;\ 3)\ \bar{d} = \frac{2}{3}\bar{e}\ ;\ 4)\ \bar{a} = 1{,}5_{(6)}\bar{e}\ .$$

Figure 7: Representation of the vector unit $\bar{e}$



Source: Adaptation from Горбов et al. (2007)

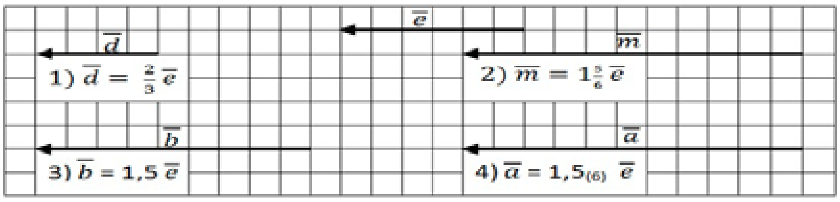
b) Build a vector opposite to the vector $\bar{b}$.

c) How to define the relation between the vector with the vector com o vetor (connected by its module and sense)?

$$-\bar{b} = \quad \bar{e};\qquad \frac{-\bar{b}}{\bar{e}} = \quad .$$

The situation *a*, as well as in the previous task, is also in the context of the construction of vectors. However, in this task, the construction of $\bar{d}$ and $\bar{d}, \bar{m}, \bar{b}$ e $\bar{a}$ (figure 8) occurs from the vector unit $\bar{e}$; that involves the numerical singularity of the rational numbers (DAVÝDOV, 1982), as well as the different numerical bases.

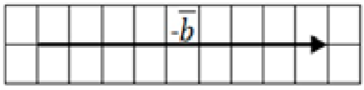
Figure 8: Representation of vectors $\overline{d}, \overline{m}, \overline{b}$ and $\overline{a}$ from the vector $\overline{e}$



Source: Resolution of the situations based on the orientation of Горбов et al. (2007).

In *b*, although it is continued with the construction of vectors, it is observed that it is approached a vector with something different in comparison to the previous ones, that is, it has an opposite direction in relation to the vector $\overline{b}$ (figure 9). The opposite of a certain vector, for Winterle (2000), is the one that keeps the same module and direction, but its sense is contrary.

Figure 9: Representation of the opposite side of the vector $\overline{b}$



Source: Adaptation based on Горбов et al. (2007)

When introducing the opposite vector is that the davydovian mode of teaching organization proposes to students the task that brings something essential to the study of negative numbers: the opposite direction of a vector in relation to another.

However, it is worth emphasizing that the introduction of these numbers only happens in the resolution of the situation *c*. Due to the fact that this situation deals with the comparison of the opposite vector $-\overline{b}$ with $\overline{e}$, it lacks the explicitation of their characteristics of module and sense. This means the appearance of another need: the specification, not only of its module, but essentially of its meaning in relation to the vector $\overline{e}$

Due to the situation *c* deal with the comparison of the opposite vector $-\overline{b}$ with the unit $\overline{e}$, it is lacked the explicitation of the characteristics of module and sense of these vectors. In other words, it s generated the specification need not only of its module in such comparison, but also of its meaning in relation to the vector $\overline{e}$.

The specification of the comparison of the vectors $-\overline{b}$ and $\overline{e}$ leads to necessary elucidation of the relation between the module and their sense and because it is the comparison of two quantities of the same species, it causes the measure of the vector $-\overline{b}$ from the unit vector $\overline{e}$. Such measurement provides that the result is a number. Therefore, it is presented the need of numerical recording of the characteristics of the module and the sense of both vectors.

In the comparison of the module and the sense of vectors $-\overline{b}$ and $\overline{e}$ the module can be expressed by the numbers until then studied in Davýdov and collaborators´teaching organization, the positive ones. However, it lacks the expression of the comparison of the sense of the vectors - and , that are opposite.

To comply with what was requested on the situation c, Горбов et al. (2006) suggest the introduction of new numbers, which express not only the relation between the modules of the vectors $-\overline{b}$ and $\overline{e}$, but also of their senses, necessarily, opposite ones

In this way, it is presented another need of representative order and of distinction, because in the historical process it is not created a sign for the new numbers. Therefore, it is necessary a symbolic component, signal, which it will add to the signs to differentiate them from those adopted for positive numbers. That is the signal that features negative numbers and is inked to the opposite senses of two vectors. This component, according to Горбов et al. (2006), corresponds to the signal minus "-" that has the purpose of representing/indicating opposite directions of two vectors.

The minus sign, therefore, fulfilled the need for representation of new numbers that have arisen from the relation between two vectors of opposing directions. As it is said byГорбов et al. (2006), the minus sign "-" indicates that the new numbers are opposite to positive.

In this way, we verified that Davýdov and collaborators attribute the significance of opposite for the negative number. That is, such authors of the proposition of teaching in analysis transfer the relation of 'opposite vectors' to 'opposite numbers' (negative), arising from the measurement of the vector opposite with the aid of the vector unit (ГОРБОВ et al., 2006).

In the context of the negative number, we inform that only after the teaching of addition and subtraction operations with negative numbers is that it will be introduced the nomenclature "negative numbers". Until then, according to Горбов et al. (2006), it is important that students understand the basic idea marked by the concept: the relation 'being opposite'. For example, the number -2 is the opposite of the number 2, and not as a proper name of a negative number, the numbers with the "minus".

This conceptual reference (the opposite of) is the basis so that, in the situation *c*, the relation of multiplicity and divisibility - between the vector $-\bar{b}$ and $\bar{e}$ the vector – be expressed in the following way: $-\bar{b} = -1{,}5\bar{b}$ *and* $\frac{-\bar{b}}{\bar{e}} = -1{,}5$. In regard to the relation of multiplicity, corresponds to the opposite vector of *b* ($-\bar{b}$) is equal to one and a half times the vector $\bar{e}$ and the sense of both is opposite. In relation to divisibility, the opposite vector of *b* ($-\bar{b}$) when divided into equal parts to the vector –results in a part and half, and also with the opposite sense

It is noted that the relations between quantities such as essential genetic basis of the concept of number, are kept for the negative, however they involve the vectors - representative of the vector quantities -, more specifically the ones in opposite senses. Therefore, they were introduced in a process whose didactics organization has favored the inter-relationship of the following ideas: the construction of the vectors, the comparison of the module and the sense between them and the comparison between those in opposite directions.

The organization of particular tasks for the appropriation of the concept of negative number with the significance of opposite, mediated by the vectorial (vector) quantity, allows for the inference that the teaching of that concept is only possible because the mentioned quantity has different characteristic from the scalar ones.

The scalar quantities do not exhibit the necessary credential to the study of the negative number, that is, the sense. Therefore, there is the emergence of an organization that creates the conditions of didactic passage from scalar quantity to the vectorial in the study of numbers, according to situation *d* from task 3. It is, therefore, from this situation that emanates conceptual needs that we will identify next.

The vector, as a genetic condition, explains the inadequacy of the scalar quantities for the study of the negative number, because they do not present an essential characteristic for this purpose: the sense. That is why the situation *d* from Task 3 is characterized as a necessity. That is, the passage from the scalar quantities to the vectorial.

The situation *e* (Task 3) also motivates the emergence of a conceptual need, by the requirement to compare, besides the modules of the vectors, their senses absent in the Scalar quantities, which are the predominant students' knowledge.

The fact of the vector opposite be absent in the context of numbers and, at the same time, constitute themselves as conceptual element of negative number, its construction (situation *b* from task 4) is transformed into a need for the introduction of the said paragraph.

In the context of the requirements identified in Tasks 3 and 4, we noted that for the study and appropriation of the concept "negative number" it was necessary to occur the passage of the comparison of scalar quantities for the Vectorial, because the negative number stems from the measurement of vectors in which one of them has the opposite sense.

Final considerations

The analysis of the four private tasks allowed us the observation that Davýdov and collaborators make use of the concepts of subtraction and equation which were learned by students in previous years to make emerge the need of the study of negative number, because the positive ones do not allow to solve some cases of subtraction and equations

In relation to figures, it is worth emphasizing that in the Davýdov and collaborators' teaching organization, they derive from the comparison/Measurement of quantities. And by such authors of that proposition of Education consider that the essence of the negative number is still in the meaning of opposite, and there was the need to perform the study of the passage from the scalar quantities to the vectorial.

This change of quantity produced another need due to requiring the observation not only of the module but also of the sense of vectors. The sense is the condition for the comparison between vectors and indication if the result will be a positive or a negative number. In the context of the demand pointed, another conceptual need concerns the presence of an opposite vector.

Beyond these, other three were identified, which emerge from the situation of the comparison of two vectors with opposite directions. In comparison it was produced the lack of specification of the module and the sense of a vector opposite in relation to another non-opposite vector. There was also a need for its numerical recording and order of representation and distinction. It is about the assignment of a signal, minus "-", to represent and differentiate the negative numbers - for which it is used the same signs as the positive. With this sign, the negatives are characterized as opposed to positive.

Based on the study of the concept of number on the davydovian proposition is always tied to its representation on the number line and the tasks analyzed do not allow to check how the introduction of negative number occurs in such a line, for a future study remains the desire to address how the negative number is inserted in the number line.

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